Correlation Specification and the Efficacy of Risk-Based Capital [a]

Chenghsien Tsai [b]
Department of Risk Management and Insurance
National Chengchi University

Ching-Syang Jack Yue
Department of Statistics
National Chengchi University

Yuan-Chih Pan
Actuarial Department
Allianz President Life Insurance Company Ltd.

Abstract

Empirical studies have shown that the risk-based capital (RBC) implemented in the insurance industries of the United States is ineffective in solvency prediction. This paper investigated how the correlation specification in obtaining Total RBC After Covariance affects the efficacy of RBC. We conducted simulations to compare the effectiveness and the efficiency of capital requirements that employ different correlation specifications. We find that alternative capital requirements commit similar type II error rates given type I error rates. Furthermore, all capital requirements demand comparable gross capital ratios to achieve target solvency probabilities. Modifying the covariance formula will not improve the efficacy of RBC therefore.

Keywords: risk-based capital, correlation, simulation
JEL Classification: G22, G28

---

[a] The authors are grateful to Martin F. Grace at Georgia State University for his constructive comments, to Georgia State University for the kind supports during the visit of the corresponding author, and to the National Science Council of Taiwan for the financial support (project number NSC 90-2416-H-004-026).
[b] Corresponding Author. No. 64, Section 2, Chihnan Road, Taipei, Taiwan, 11623, R.O.C.;
Tel: +886-2-2936-9647; Fax: +886-2-2939-3864; e-mail: ctsai@nccu.edu.tw.

1. Introduction

The capital requirements for the insurers in the United States, Japan, Taiwan, and several other countries are risk-based capital (RBC). RBC was first developed by the National Association of Insurance Commissioners (NAIC) in response to the concern of the U.S. Congress about the adequacy and accuracy of insurance solvency surveillance. However, the effectiveness of RBC as an early warning tool has been called into question by empirical studies. Cummins, Harrington, and Klein (1995) analyzed the ability of RBC to predict the solvencies/insolvencies of property-casualty (P/C) insurers and found that the predictive accuracy of the RBC ratio was low. Grace, Harrington, and Klein (1998) also found that few P/C companies that later failed had the RBC ratios within the NAIC's ranges for regulatory actions. They further found that RBC provided inferior predictive power when compared to the Financial Analysis and Surveillance Tracking (FAST) audit ratio system. Cummins, Grace, and Phillips (1999) confirmed that the RBC ratio provided poor discriminatory accuracy and added little information to FAST. They further showed that the cash flow simulation variables added significant predictive power to the model containing both RBC and FAST. Pottier and Sommer (2002) confirmed again that RBC was a poor discriminatory tool. The risk measure produced by A. M. Best, Capital Adequacy Relativity Ratios, had better predictive abilities than RBC. Their results also demonstrated that two of the overall risk measures (FAST scores and Best's rating) were superior in predictive ability to risk-based capital measures. The literature documented clearly that RBC was ineffective in indicating an insurer's solvency.

The incapability of RBC in predicting solvency/insolvency may be due to four reasons.[1] First, the risk charges applied to various assets, liabilities, and businesses can be imprecise and may not be accurate meters for the risks to be measured. For instance, the charges to the positions in bonds with different qualities and maturities might not accurately reflect the underlying credit risk.[2] Second, the imposed correlation specification is wrong. The current formula to obtain Total RBC After Covariance for property-casualty insurers, $R0+\sqrt{R1^2+R2^2+R3^2+R4^2+R5^2}$, implicitly assumes that R1 through R5 are not correlated with each other while the sum of these risks is perfectly and positively correlated with R0. The assumption that risks are either perfectly correlated or not correlated apparently deviates from the real world case and may significantly distort the risk measurement. Third, RBC is a local-valuation method instead of a full-valuation method. RBC is fundamentally linear since the potential loss in a portfolio's value V is computed as $\Delta V = V_0 \times \beta_0 \times \Delta P$, where β_0 denotes the portfolio's sensitivity to changes in prices evaluated at the current position V_0 and ΔP denotes the potential price changes.[3] Linear approximation is valid only for a narrow range of price movements, whereas insolvencies

[1] These four reasons are directly related to the formula and/or methodology of RBC. There exist some indirect or exogenous reasons such as the lack of qualitative adjustments (Pottier and Sommer, 2002) and the regulatory arbitrage exploited by insurers based on the transparency of RBC.

[2] Wrong risk charges may also result from the inadequacy classifications of risks and positions.

[3] Hence, the risk charges in RBC = $\beta_0 \times \Delta P$.

usually involve large changes in asset and/or liability values. Finally, RBC is static in nature rather than dynamic. It profiles the risk of a company based mainly on a snap shot of the company at a given point of time without considering the dynamic relations among asset and liability values over time. Static analyses also ignore the interactions between outside market conditions and internal managers' actions/reactions that are usually critical to an insurer's solvency/insolvency.

In this paper we investigate the impact of the correlation specification in obtaining Total RBC After Covariance on the efficacy of RBC. Cummins, Grace, and Phillips (1999) demonstrated that a dynamic cash flow model with the full-valuation method outperformed static RBC that employed the local-valuation method in solvency/insolvency predictions. The results of Pottier and Sommer (2002) implied to some extent that alternative risk charges to RBC could perform better. We are the first to examine the potential ineffectiveness of RBC caused by the correlation specification. Since the risk of a portfolio depends not only on the variances of individual components but also on the covariances among individual components, mis-specifying correlations will lead to a wrong estimation about the portfolio risk. Our goal in this paper is to assess the adverse impact of the correlation mis-specification on the efficacy of RBC.

To accomplish our goal, we conducted simulations to compare the effectiveness and the efficiency of alternative capital requirements that employ assorted correlation specifications. We first constructed a hypothetical world in which equity investment risk, interest rate risk, and underwriting risk correlate with each other to different degrees. Then we calculated several capital requirements for a representative property-casualty insurer based on the simulated data (paths). These capital requirements have the same risk charges to the insurer's activities but have different correlation specifications. The tested specifications include independence, perfectly positive/negative correlations, and "empirical" correlations estimated from simulated "historical" paths. As the simulation keeps going, the risky positions of the insurer as well as the corresponding required capital evolve and the insurer may become insolvent. We then compared the effectiveness and efficiency of these capital requirements in terms of their solvency predicting accuracies and the minimal average capital ratios to achieve target solvency probabilities respectively.

Surprisingly, we found that the correlation specification affects neither the effectiveness nor the efficiency of the capital requirements. The capital requirements using different correlation specifications have equivalent type I error rates given type II error rates. These capital requirements are therefore equally effective in predicting an insurer's solvency/insolvency. Furthermore, all capital requirements demand comparable average capital ratios to achieve target solvency probabilities. They are similarly efficient, in other words. We further tried different simulation settings and confirmed the robustness of our results.

The main reason for the above findings is that the capital requirements employing distinct correlation specifications are highly correlated with each other. The correlation coefficients are all above 0.9. These requirements, albeit applying different correlation specifications, are like multiples of one another. The solvency rankings of insurers are thus quite similar across capital requirements, even though the required amounts do differ from each other. Since the correlation specification does not alter the rankings of the insurers, it does not affect the type I error rates and the minimal average capital ratios given type II error rates and target solvency probabilities respectively. Therefore, the correlation specification has no impact on the effectiveness and the efficiency of RBC.

Our results imply that revising the current correlation specification will not improve the efficacy of RBC in predicting solvencies/insolvencies. Therefore, the adjustment that NAIC made to incorporate the correlation between common stocks and other assets in the early 2000s would probably be in vain in this aspect. Regulators need to modify the risk factors, switch to a full-valuation method, and/or employ a dynamic analysis tool in order to enhance the effectiveness and efficiency of the current capital requirement system. Our results have similar implications for the Basel II Accord that sets up the capital requirements for banks. Changing the current correlation specification will not alter the efficacy of the Basel II Accord.[4] Adopting the internal model methods that lean towards full valuation

[4] Basel II Accord implicitly assumes that credit risk, market risk, and operational risk are perfectly and positively correlated with each other since it simply adds up the required capital of individual components.

method, dynamic approach, and more accurate risk charges is the way to go.

The remainder of this paper is organized as follows. In section 2 we describe the stochastic processes of risks, the representative insurer, the simulation structure, the specification of capital requirements, and the criteria to evaluate the effectiveness and the efficiency of capital requirements. Section 3 contains the simulation results and our analyses of the efficacy of alternative capital requirements, and at the end provides a simple experiment to demonstrate our rationale for the results. We draw our conclusions in the fourth and final section.

2. Simulation Setting

In this section, we first build up a hypothetical world involving three types of risk: equity investment risk, interest rate risk, and underwriting risk.[5] Assuming that the stock price follows a geometric Brownian motion, the instantaneous spot rate follows the Cox, Ingersoll, and Ross (1985) model, and the loss ratio has a normal distribution, we simulate sets of paths for stock prices, bond prices, and loss ratios while allowing for correlations among risks. We then construct a representative insurer based roughly on the asset allocation (in bonds and stocks), capital structure (premiums to surplus ratio), loss development pattern, expense ratio, and growth of the U.S. property-casualty insurance industry. Next, we calculate four capital requirements for the insurer using a linear risk

[5] These three types of risks are arguably the most significant risks faced by a property-casualty insurer. We deal with more types of risks later in section 3.4.

measuring method similar to RBC. These four requirements differ from each other only in the assumption about the correlations among risks. As the financial positions of the insurer evolve in the simulation, we keep calculating the corresponding capital requirements. Finally, we assess the performance of these requirements. We use the type I error rate given a type II error rate to measure the effectiveness of RBC as an early warning tool and the minimal average capital ratio given a target solvency probability to measure the efficiency of RBC as a capital requirement scheme

2.1. Underlying Risks

To model the equity investment risk, we assume that the index of the stock market, *S*, follows the geometric Brownian motion:

$$dS = \mu S dt + \sigma S dz\,, \tag{1}$$

where μ denotes the expected return of the stock index per annum with continuous compounding, σ is the volatility of the index per annum, and *z* follows a Wiener process. With regard to the interest rate risk, we assume that the risk-neutral process for the short-term interest rate, *r*, is

$$dr = q(m - r)dt + v\sqrt{r}dz\,, \tag{2}$$

where *m* represents the long-run average of the short rate, *q* reflects the speed of mean reversion, and *v* is the volatility parameter of the process. For the underwriting risk we assume that the loss ratio is a random variable having a normal distribution. The parameters for the above three models are specified as follows.[6]

Model Parameters			
Stock Index	μ = 12%	σ = 24%	
Interest Rate	m = 6%	q = 10%	v = 2%
Loss Ratio	mean = 75%	standard deviation = 25%	

The correlations among equity investment risk, interest rate risk, and underwriting risk are assumed to be as per the following matrix.[7]

	Stock Market Returns	Changes in Short Rates (Δr)	Loss Ratios
Stock Market Returns	1	0.5	-0.5
Δr	0.5	1	-0.5
Loss Ratios	-0.5	-0.5	1

This correlation matrix means that stock returns are positively correlated with changes in interest rates and it therefore implies that stock returns are negatively correlated with the investment returns of bonds. It also indicates that stock returns and loss payments have a negative correlation coefficient. Furthermore, the correlation matrix implies that bond returns are positively correlated with losses.

[6] The starting value of the short rate is 6%.

[7] The correlation matrix is designed to reflect mid-range correlations. The matrixes that represent high and low correlations are considered in section 3.3.

The risk models and correlation matrix described above represent the underlying risk structure of our hypothetical world. We use them to simulate sets of stock index paths, short rate paths, and loss ratio paths. The insurer's financial positions, including the values of invested stocks, the values of bonds owned, and loss payments, are determined by these sets of simulated paths. The mappings from stock indices to stock values and from loss ratios to loss payments are straightforward. The mapping from short rates to bond prices is through Cox, Ingersoll, and Ross (1985): the price at time t of a default-free zero-coupon bond that pays \$1 at time T equals

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)}, \tag{3}$$

where $B(t,T) = \dfrac{1-e^{-q(T-t)}}{q}$ and $A(t,T) = \exp[\dfrac{(B(t,T)-T+t)(q^2 m - \frac{v^2}{2})}{q^2} - \dfrac{v^2 B(t,T)^2}{4q}]$.

The correlation matrix is incorporated in the simulation through the Cholesky factorization. Suppose that we want to transform an N vector η that is composed of independent variables all with unit variances to a vector of N values of ε that displays some correlation structure R. More specifically, $V(\varepsilon) = E(\varepsilon\varepsilon') = R$ and $V(\eta) = I$ where I is the identity matrix. Since R is a symmetrical real matrix, it can be decomposed into its Cholesky factors: $R = TT'$, where T is a lower triangular matrix with zeros in the upper right corners. Then $\varepsilon = T\eta$. In other words, we obtain the random number vector incorporating correlations, ε, by multiplying an independent random number vector η with the lower triangular matrix T decomposed from the correlation matrix R.

The underlying risk structures, including risk models and correlations, are not observable to regulators. More specifically, regulators are assumed to be able to observe only simulated stock market returns, interest rates, and loss ratios. They know neither the underlying stochastic processes nor the underlying correlation structure. Therefore, they have to estimate/speculate the volatilities of individual risks and the correlations among risks when establishing capital requirements, which is consistent with the real world case.

2.2. The Representative Insurer

Suppose that a newly established property-casualty insurer with a surplus of 65 million dollars starts to underwrite businesses and receive premiums of 100 million dollars in cash at the beginning of year 1. To underwrite these businesses, the insurer incurs and pays the underwriting expenses in cash upfront. The underwriting expense ratio is assumed to be 25%. The remaining cash and the initial surplus are then invested in the stock index and Treasury Strips with weights of 35% and 65% respectively. The maturity of invested bonds ranges from one year to fifteen years with the following proportions:

Maturity	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Proportion (%)	3	3	4	5	10	10	10	10	10	5	5	5	5	10	5

Assuming that the fair value of the reserves is equal to premiums written net of expenses, we get the following balance sheet for the representative insurer at the beginning of year 1:

(millions of dollars)

Assets		Liabilities and Surplus	
Stocks	**$49**	**Liabilities**	**$75**
Treasury Strips	**$91**		
1-Year	$2.73		
2-Year	$2.73		
3-Year	$3.64		
4-Year	$4.55		
5-Year	$9.1		
6-Year	$9.1		
7-Year	$9.1	**Surplus**	**$65**
8-Year	$9.1		
9-Year	$9.1		
10-Year	$4.55		
11-Year	$4.55		
12-Year	$4.55		
13-Year	$4.55		
14-Year	$9.1		
15-Year	$4.55		
Total Assets	**$140**	**Total Liabilities and Surplus**	**$140**

At the end of year 1, investment returns and loss ratios are realized according to the simulation using the risk models and correlation matrix described in section 2.1. To account for loss development and business growth, we assume that the insurer's businesses grow 5% annually and have a ten-year development period with the loss development function *D(t)* as follows:

t	1	2	3	4	5	6	7	8	9	10
$D(t)$ (%)	50	30	10	5	3	1	0.5	0.3	0.1	0.1

Simulated loss ratios represent 95.93% of the ultimate loss divided by the premiums written, where the ultimate losses for the businesses written in any given year are defined as the total payments across all development years paid for the written businesses[8]. More specifically, the ultimate losses for the businesses written in year *i* equal

$$\frac{\text{Simulated Loss Ratio} \times \text{Premiums Written in Year i}}{0.9593}. \tag{4}$$

The loss payment in development year *t* for the business written in year *i* is then equal to the *i*th year's ultimate losses times *D(t)*.

The insurer pays losses first with the bonds that mature at this time, and then with the sales of bonds and stocks. To keep the asset allocation of the insurer intact with loss payments, the insurer sells all assets proportionally. Specifically, we assume that the insurer sells each type of invested assets, including stocks and bonds of different maturities, with the proportion of

[8] The adjustment factor, 0.9593, is to account for the effect of growth and time value of money. For an insurer that does not have growth in premiums written, calendar-year loss ratios are equal to the ratio of the ultimate losses to the premiums written. For a growing insurer, however, calendar-year loss ratios will be less than the ratio of the ultimate losses to the premiums written because the denominators of loss ratios grow with time. Furthermore, time value of money should be considered since loss payments span a 10-year period. After a simple spreadsheet work, we obtain the adjustment factor of 0.9593 for a growth rate of 5% and a discount rate of 7%.

$$\frac{\text{Losses} - \text{Face Amount of Matured Bonds}}{\text{Market Value of Total Assets Excluding Matured Bonds}}. \tag{5}$$

We then deduct the amount of losses paid from the reserves and attain the year-end balance sheet for year 1.[9]

At the beginning of year 2, the insurer underwrites $105 million (=$100*(1+5%)) businesses, pays 25% of underwriting expenses, and invests 35% and 65% of the net amount in stocks and bonds respectively. Our simulation model then generates investment returns and loss ratios for year 2. The insurer pays losses with matured bonds and/or with the proportional sales of stocks and bonds at the end of year 2. Reserves are reduced by loss payments, and we obtain the year-end balance sheet for year 2 thereafter. Similar procedures are repeated for twenty-five years, or, repeated until the insurer becomes insolvent.[10] The insurer is deemed insolvent whenever its surplus, the difference between the market value of assets and the fair value of reserves, is smaller than zero.

The parameters of the underlying risk models and the representative insurer are chosen so that the insurer has an "adequate" insolvent probability of about 5% to facilitate subsequent analyses. We tried various sets of parameters and learned that the key variables to the insolvent probability are the initial premium-to-surplus ratio, sum of the expense ratio

[9] Reserves might be smaller than loss payments in extreme cases. We set reserves as zero in these cases and deduct the deficit from the surplus.

[10] The compositions of the bond portfolio held by the insurer soon become complicated because the maturity of a bond will be reduced along with the simulation time while the new invested bonds keep coming in.

and expected loss ratio, growth rate, returns of investments, and volatilities of risks. As expected, higher leverage ratios, combined ratios, and/or volatilities of risks result in higher insolvent probabilities while higher expected investment returns lead to fewer bankruptcies.

2.3. Capital Requirements

We calculated four capital requirements using the delta-normal method of value at risk (VaR) which is similar to the methodology of RBC.[11] The general assumption behind the delta-normal method is that underlying risk factors (e.g., stock returns and changes in interest rates) follow normal distributions. This method is called "delta" because it is a local valuation method using first derivatives to approximate potential losses. This method of VaR is similar to that of RBC because both estimate potential losses in the form of risky positions times the coefficients in which the coefficients indicate the riskiness of the positions.

To cover the potential losses of fixed-income securities in a year, all capital requirements ask for:

$$\text{Market Position in Bonds} \times \alpha \times \sigma(\Delta r) \times \text{Modified Duration of Bond Portfolio}, \qquad (6)$$ [12]

where α is chosen by regulators to achieve the desirable coverage probability and $\sigma(\Delta r)$

[11] Interested readers can refer to Chapter 10 in Saunders and Cornett (2006) and/or Chapter 11 in Jorion (2000) for more details about the delta-normal method.

[12] The modified duration term is used to map changes in interest rates to changes in bond prices. The product, Modified Duration of Bond Portfolio $\times \alpha \times \sigma(\Delta r)$, can be deemed as the risk charge in RBC.

denotes the estimated standard deviation of Δr per annum. Similarly, the required capital for the potential losses of the stock index in a year is:

$$\text{Market Position in the Stock Market} \times \alpha \times \sigma(\text{stock market return}). \tag{7}$$

Finally, the capital requirements demand

$$\text{Reserve} \times \alpha \times \sigma(\text{loss ratio}) \tag{8}$$

to cover the one-year underwriting risk. We assumed that the regulators designate α as 2.326 to cover 99% of the potential losses same as the Basel Committee does, and that they estimate the standard deviations of the stock market returns, Δr, and loss ratios using the most recent fifteen-year data.[13]

Departing from the above common formulas to cover individual risks, the four capital requirements specify different correlations among risks in estimating a portfolio risk. Let R_m denote the correlation matrix among the stock market returns, Δr, and loss ratios used in the m-th capital requirement ($m = 1 - 4$). We design R_1, R_2, R_3, and R_4 as

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}, \text{and } \begin{bmatrix} 1 & emp_{sr} & emp_{sl} \\ emp_{rs} & 1 & emp_{rl} \\ emp_{ls} & emp_{lr} & 1 \end{bmatrix} \text{ respectively,}$$

where emp_{ij} represents the correlation coefficient between risk factors i and j estimated using the past fifteen-year data. The current RBC employs R_1 as the correlation matrix in the formula, whereas the delta-normal method of VaR commonly uses R_4. R_2 is motivated by

[13] Indeed, the choice of α does not matter. The reason will become apparent in section 3.4.

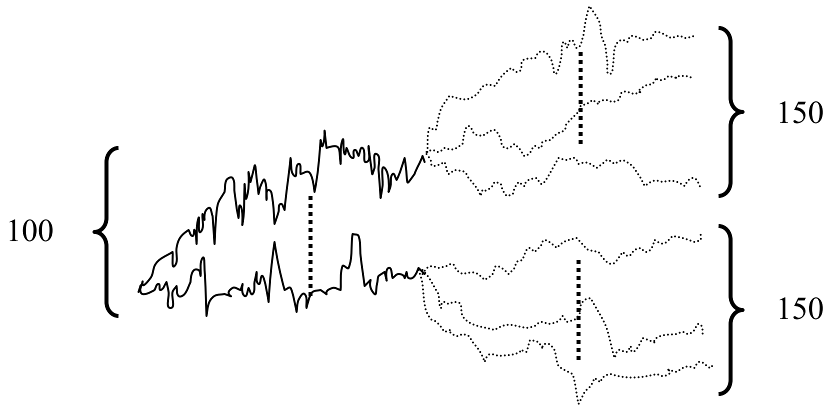
the observation that the stock market usually booms with rising interest rates. R_3 is a contrasting example of R_2 in that the former assumes perfect correlations between financial markets and insurance markets whereas the latter assumes independence.

Let S be the diagonal matrix with estimated σ *(stock market return)*, estimated σ *(Δr) $\times$ modified duration of bond portfolio*, and estimated σ *(loss ratio)* on its diagonal. Further, let x denote the vector representing the dollar market values of the positions in stocks, bonds, and reserves. The aggregate required capital set by the m-th capital requirement at the beginning of a year to cover the potential losses of the insurer in that year is then equal to:

$$2.326 \times \sqrt{x' S' R_m S x}\,. \tag{9}$$

2.4. The Simulation Structure

We first simulated 100 sixteen-year paths for stock market returns, interest rates, and loss ratios prior to the establishment of the representative insurer so that regulators have "historical" data to estimate the risk charges in the capital requirements to begin with. Each set of paths will result in an S of Equation (9). The insurer therefore has 100 initial capital requirements corresponding to the 100 simulated market histories. For each set of paths, we further simulated 150 sets of twenty-six-year paths that represent future market conditions. Our simulation structure looks like the following:



The total of simulated paths is therefore 15,000.

2.5. Efficiency and Effectiveness Criteria

We assessed the efficiency and effectiveness of alternative capital requirements with two criteria respectively. The efficiency assessment is from Dimson and Marsh (1997). Define F_{ms} to be the average value of the capital requirements expressed as a percentage of the insurer's gross value under the capital requirement method m (m = 1 – 4) to achieve a given solvency rate s (s = 100%, 99%, 98%, …, 1%). More specifically,

$$F_{ms} = \frac{1}{\sum_{i=1}^{10,000} p_i} \Sigma_{i=1}^{10,000} \Sigma_{t=0}^{p_i} \frac{k_{ms}^{*} \times C_{mit}}{G_{it}}, \tag{10}$$

where C_{mit} is the capital requirement calculated using method m at time t (t = 0, 1, 2, …, p_i) in path i (i = 1, 2, 3, …, 10,000), p_i = min [26, the time when the insurer becomes insolvent in path i], G_{it} is the gross value (the value of assets plus the value of liabilities) of the insurer at time t in path i, and k_{ms}^{*} is the minimal multiplier (safety factor) needed to provide the coverage rate s under method m. Given an s, a lower F_{ms} means a more efficient capital requirement since the requirement demands a lower capital ratio on average for the same solvency probability.

The effectiveness criterion is the prediction accuracy of the capital requirements. A capital requirement is seen as sending an early warning signal whenever the insurer's surplus is smaller than the requirement. A capital requirement incurs a type II error when it hits a false alarm, i.e., when it sends out a warning message but the insurer remains solvent after a year. On the other hand, a capital requirement incurs a type I error when it does not signify a warning but the insurer turns out to be insolvent at the end of the year. Since type I and type II error rates have a trade-off relation, we have to fix one of them and then compare the other when assessing the prediction accuracy of alternative capital requirements. We choose the type I error rate as the anchor because it is more of a concern to regulators than the type II error. Therefore, our way to assess the effectiveness of a capital requirement is the type II error rate given a type I error rate. A lower type II error rate given a type I error rate means a more accurate prediction and thus a more effective capital requirement. To achieve the given type I error rates 1-*s*, we multiply the calculated capital requirements by k_{ms}^{*} and then examine the resulting type II error rates.

3. Results and Analyses

3.1. Results about Effectiveness

Surprisingly, we find from Figure 1 that the four capital requirements employing different correlation specifications seem to be comparably effective. The numbers in Table

1 report that they all commit similar type II errors for most type I error rates. For instance, the type II error rates of the four capital requirements range between 2.6% to 3% when regulators allow a 4% of type I error rate. If regulators are willing to tolerate higher type I error rates, then the differences in type II error rates among the capital requirements will be even smaller.

[Insert Figure 1 and Table 1 Here]

The fourth capital requirement that employs the correlation matrix estimated from historical data seems to be the least effective. For type I error rates smaller than 30%, this capital requirement commits the highest type II error rates. The differences in type II error rates become noteworthy as regulators require very low type I error rates such as 0% or 1%. The inferiority of the fourth capital requirement might be due to the error in estimating correlations. The standard deviations of the estimated correlations in the simulations are about 0.5. The large estimation error resulting from the usage of the past 15-year annual data may have impaired the effectiveness of the capital requirement. More generally, the benefit of incorporating correlation estimation into capital requirements with only annual data might be outweighed by the cost of accompanied estimation errors.

To see whether the effectiveness of the four capital requirements differ from each other significantly, we follow Cummins, Grace, and Phillips (1999) in conducting the receiver operating characteristic (ROC) analysis. The goal of the ROC analysis is to test whether a model outperforms another in classifying observations into two mutually exclusive groups. The analysis is done over the area below the ROC curve. A ROC curve is two dimensional, with the type II error rate being the x-axis and the complement of the type I error rate (1 – the type I error rate) as the y-axis. The area below the ROC curve indicates the accuracy of a model in a binary prediction exercise. A model that perfectly discriminates between the insolvent and solvent insurers will have an area of 1 while a model with no discriminatory power will have an area of 0.5. The ROC curves of the four capital requirements employing different correlation specifications are shown in Figure 2.

[Insert Figure 2 Here]

Although the ROC curves of the four capital requirements are close to each other, we cannot conclude that the effectiveness of these four requirements is statistically equivalent without first performing some statistical tests. To test the null hypothesis of equal areas under any two ROC curves, we calculate

$$z = \frac{A_1 - A_2}{\sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}}, \tag{11}$$

where A_i stands for the area under ROC curve i (i = 1, 2), σ_i is the standard error of A_i, and ρ denotes the correlation coefficient between A_1 and A_2. The standard error and correlation are estimated using the bootstrapping results generated from repeating 1,000 times the entire simulation.[14] The test statistic z is a standard normal variable, and a large value of z

[14] More specifically, we performed 1,000 times the simulation described in section 2 to obtain 1,000 sets of

indicates the rejection of the null hypothesis. We find that the *z* values calculated for pairs of the four capital requirements are all smaller than 1. Therefore, we cannot reject the null hypothesis that the ROC curves of the four capital requirements are the same.

3.2. Results about Efficiency

Similarly, we find from Figure 3 that the four capital requirements employing different correlation specifications seem to be comparably efficient. The numbers in Table 2 report that on average they all demand similar levels of gross capital ratios to achieve the target solvency probabilities. For instance, the minimal average capital ratios required under alternative correlation specifications center around 17% when regulators demand a 96% of solvency rate. The differences in the required capital ratios are indeed all smaller than 1.5%, except in the case of zero tolerance for insolvency.

[Insert Figure 3 and Table 2 Here]

We also performed a test like the one in section 3.1 to see whether the areas under the four capital ratio curves are statistically the same. The z tests done on the simulation results using the bootstrapping report that z values are all smaller than 1, which implies that we cannot reject the null hypothesis that the minimal average capital ratio curves of the four capital requirements are the same. Therefore, we conclude that the efficiency of these four requirements is statistically equivalent.

ROC curves. We then estimated the standard error of the area under each curve and the correlation coefficients among the four areas from this time-consuming bootstrapping.

3.3. Robustness Check

To make sure that the underlying correlation structure of our hypothetical world does not bias against any one of the correlation specifications used in capital requirements, we tried two additional structures:

Low Correlation Structure

	Stock Market Returns	Changes in Short Rates (Δr)	Loss Ratios
Stock Market Returns	1	0.1	-0.1
Δr	0.1	1	-0.1
Loss Ratios	-0.1	-0.1	1

and

High Correlation Structure

	Stock Market Returns	Changes in Short Rates (Δr)	Loss Ratios
Stock Market Returns	1	-0.9	0.8
Δr	-0.9	1	-0.7
Loss Ratios	0.8	-0.7	1

Holding everything else in the simulation intact, we obtained two additional sets of results. Parts of these results are displayed in Table 3 through Table 6 and Figure 4 to Figure 7. The tests on the simulation results using bootstrapping confirm our previous findings. The effectiveness and efficiency of capital requirements using alternative correlation specifications are equivalent in these two cases as well. Our findings are thus robust across the underlying correlation structure of the simulated world.

[Insert Tables 3 - 6 and Figures 4 - 7 Here]

We also tried several other sets of parameters for the underlying risk models and the representative insurer, e.g., the expected returns of investment, mean loss ratio, volatilities of risks, initial premium-to-surplus ratio, expense ratio, and growth rate. The insolvency probability of the insurer does differ across parameter sets, but the effectiveness and efficiency of alternative capital requirements remains equivalent. We further tried other simulation structures of 1×10,000 and 10,000×1, and confirmed again the validity of our findings. We even tried different interest rate models, different asset allocations of the insurer, and different loss development function. In all circumstances our proposition holds.

3.4. An Experiment

To further secure the robustness of our results, we conducted a simulation experiment. We first created six random variables with normal distributions. The means and standard deviations of the six normal distributions were arbitrarily chosen to be as follows.

Variable	*X1*	*X2*	*X3*	*X4*	*X5*	*X6*
Mean	170,000	5,000,000	250,000	-400,000	-100,000	-1,000,000
Standard Deviation	140,000	500,000	370,000	310,000	20,000	750,000

The correlations of these variables are randomly generated by the uniform distribution [-1, 1]. Then we calculated $\sqrt{X'IX}$ and $\sqrt{X'R_{rand}X}$ in which *X'* denotes the vector [*X1 X2 X3 X4 X5 X6*], *I* denotes the 6×6 identity matrix, and R_{rand} is the 6×6 correlation matrix. $\sqrt{X'IX}$ can be deemed as the risk of a portfolio consisting of three assets and three liabilities when assets and liabilities are independent of each other; $\sqrt{X' R_{rand} X}$ represents the portfolio risk considering correlations. We generated 3,000 X for each R_{rand} and calculated the correlation coefficient ρ between $\sqrt{X'IX}$ and $\sqrt{X' R_{rand} X}$. After generating 1,000 R_{rand}, we estimated the mean and standard deviation of ρ.

We found that $\sqrt{X'IX}$ and $\sqrt{X' R_{rand} X}$ are highly correlated regardless of R_{rand}. The mean and median of ρ are 0.72 and 0.78 respectively, with a small standard deviation of 0.17. This experiment implies that no matter how regulators specify the correlations among R0 through R5, the specification will have similar impact on all insurers' Total RBC After Covariance. In other words, most insurers' total RBC will be adjusted upwards or downwards at the same time without altering the ranking of the insurers' RBC. Since the correlation specification does not change the solvency ranking, it will not affect the effectiveness and efficiency of RBC.

It must be noted that our argument is not that different correlation specifications will result in similar required capital amounts. Neither do we argue that alternative capital requirements will result in similar pairs of type I and type II errors. The capital requirement that assumes perfectly positive correlations among assets and perfectly negative correlations between assets and liabilities is the most stringent one and thus will commit the smallest type I error rate. On the other hand, the requirement assuming independence will demand the least capital and commit the highest type I error. The results in Table 7 are consistent with our expectation. For instance, the type I error rate of the capital requirement using R_3 is 0.0038 when $\alpha = 1.645$ whereas the rate for the case of using R_1 is 0.0301 given the same α.

[Insert Table 7 Here]

What we are arguing is that the correlation specification does not alter the relative capital adequacy among insurers. An insurer with a high/low RBC ratio (defined as 0.5* Total RBC After Covariance/Total Adjusted Capital) under one correlation specification most likely will have a high/low RBC ratio under another specification. Since its financial strength ranked in the industry is not altered by the correlation specification, the specification will not affect the effectiveness and efficiency of capital requirements. More specifically, $\frac{C_{mit}}{G_{it}}$ are highly correlated across m in Equation (10) and are thus like multiples of each other with similar rankings in gross capital ratio. On average the step of searching for k^*_{ms} will offset the differences in $\frac{C_{mit}}{G_{it}}$ and result in similar F_{ms} and type II error rates.[15] For instance, the facts that $\frac{C_{3it}}{G_{it}}$ and $\frac{C_{1it}}{G_{it}}$ are highly correlated and that R_3 is more stringent than R_1 lead to the results that the insurer's capital ratios are similarly ranked across correlation specifications and that k^*_{1s} is larger than k^*_{3s}. The results then counterbalance the difference in stringency. Therefore, different correlation specifications will not affect the effectiveness and efficiency of capital requirements.

[15] The effect of searching for k^*_{ms} is equivalent to choosing an appropriate α. A smaller α will result in a larger k^*_{ms}. They are substitutes/compliments to each other. The choice of α alone therefore will not affect the efficacy of capital requirements.

4. Conclusions

Countries such as the United States, Canada, Japan, Singapore, and Taiwan have adopted RBC as the capital requirements for insurance industries. Empirical studies using the U.S. property-casualty insurers' experiences however show that RBC is ineffective in predicting an insurer's solvency/insolvency. The reasons for the ineffectiveness can be inaccurate risk charges, wrong correlation specifications, the usage of local valuation methodology, and the static nature of RBC. Cummins, Grace, and Phillips (1999) documented that a cash flow model that uses the full valuation method and is more dynamic than RBC has a stronger performance than RBC in providing accurate early warnings. Sommer and Pottier (2002) suggested the potential improvement that more accurate risk charges can provide for RBC.

This paper investigated how the correlation specification in obtaining Total RBC After Covariance affects the efficacy of RBC. We first constructed a hypothetical world in which a representative insurer faces stock market risk, interest rate risk, and underwriting risk. We then designed four capital requirements that employed the linear local valuation method with different correlation specifications. Since the insurer's capital requirements and solvency status change along with the simulation, the simulated outcomes enabled us to examine how correlation specifications affect the effectiveness and efficiency of alternative capital requirements.

To our surprise we found that the correlation specification does not affect the efficacy of capital requirements in terms of their solvency predicting accuracies and demanded capital ratios for target solvency probabilities. All capital requirements have comparable type II error rates, given type I error rates. Also, all four requirements demand equivalent gross capital ratios on average given target solvency probabilities. The main reason for these results is that capital requirements employing different correlation specifications are highly correlated with each other. The correlation specification therefore does not alter the solvency ranking among insurers and results in similar capital requirement efficacy.

Our results imply that modifying the covariance formula alone will not improve the efficacy of RBC and that Basel II. Revising all correlation specifications in RBC and Basel II, including the correlations within each risk component/category, may enhance the materiality of the correlations. It will however introduce significant estimation errors since regulators inevitably have to use low frequency data to estimate a large correlation matrix. Insurance regulators as well as the regulators in other financial services industries therefore should take different routes to improve current capital requirements, e.g., fine tuning risk charges or move towards a dynamic full-valuation method.

References

Cummins, J. D., Grace, M. F., Phillips, R. D., 1999. Regulatory solvency prediction in property-liability insurance: risk-based capital, audit ratios, and cash-flow simulation. Journal of Risk and Insurance 66, 417--458.

Cummins, J. D., Harrington, S. E., Klein, R., 1995. Insolvency experience, risk-based capital, and prompt corrective action in property-liability insurance. Journal of Banking and Finance 19, 511--527.

Cox, J. C., Ingersoll, J. E., Ross, S. A., 1985. A theory of the term structure of interest rates. Econometrica 53, 385--408.

Dimson, E., Marsh, P., 1997. Stress tests of capital requirements. Journal of Banking and Finance 21, 1515--1546.

Grace, M. F., Harrington, S. E., Klein, R., 1998. Risk-based capital and solvency screening in property-liability insurance: hypotheses and empirical tests. Journal of Risk and Insurance 65, 213--243.

Jorion, P., 2000. Value at Risk: The New Benchmark for Managing Financial. McGraw-Hill, Taipei.

Pottier, S. W., Sommer, D. W., 2002. The effectiveness of public and private sector summary risk measures in predicting insurer insolvencies. Journal of Financial Services Research 21, 101--116.

Saunders, A., Cornett, M. M., 2006. Financial Institutions Management: A Risk Management Approach, 5th edition. McGraw-Hill, Taipei.

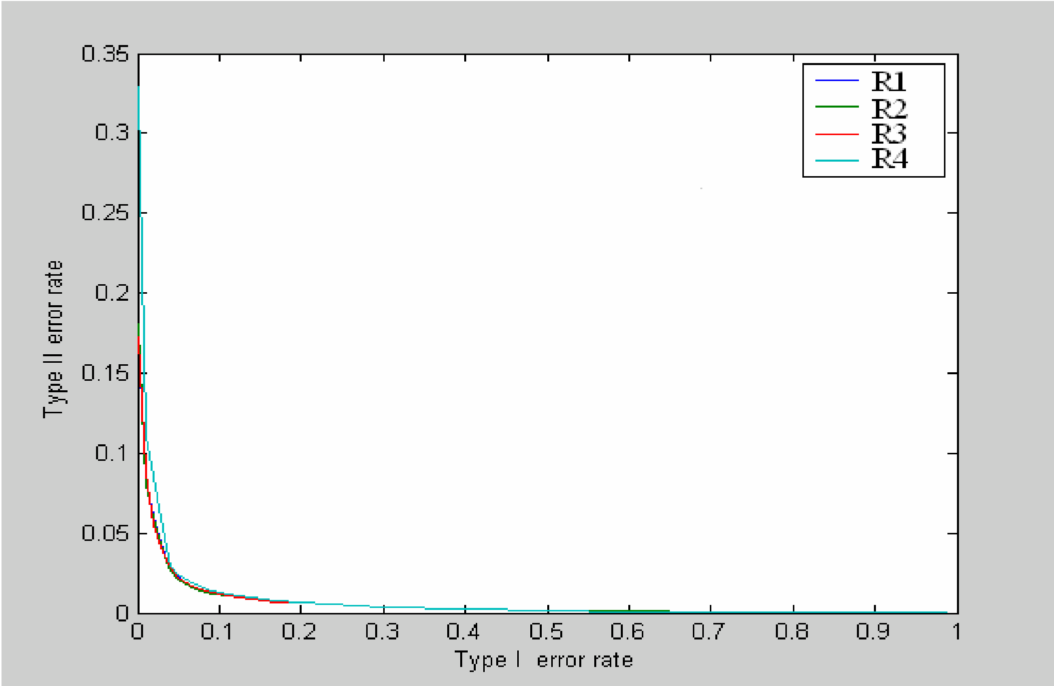


Figure 1: The type II error rates of capital requirements using alternative correlation specifications given type I error rates

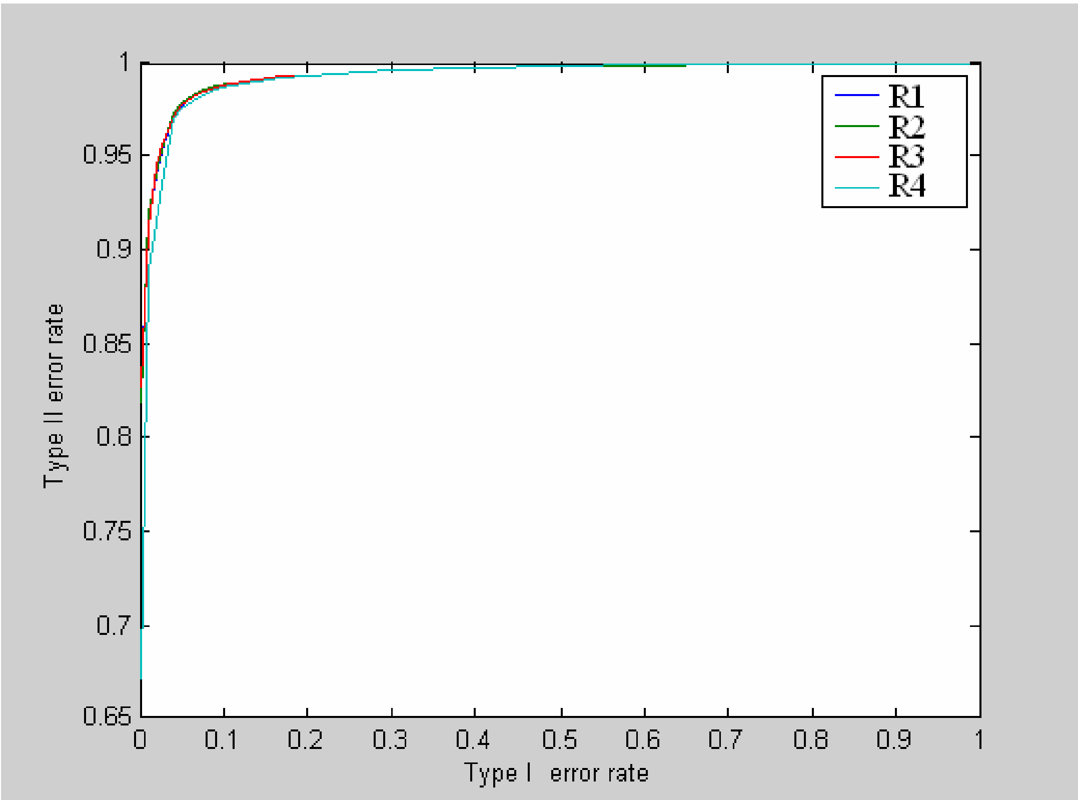


Figure 2: The ROC curves of capital requirements using alternative correlation specifications

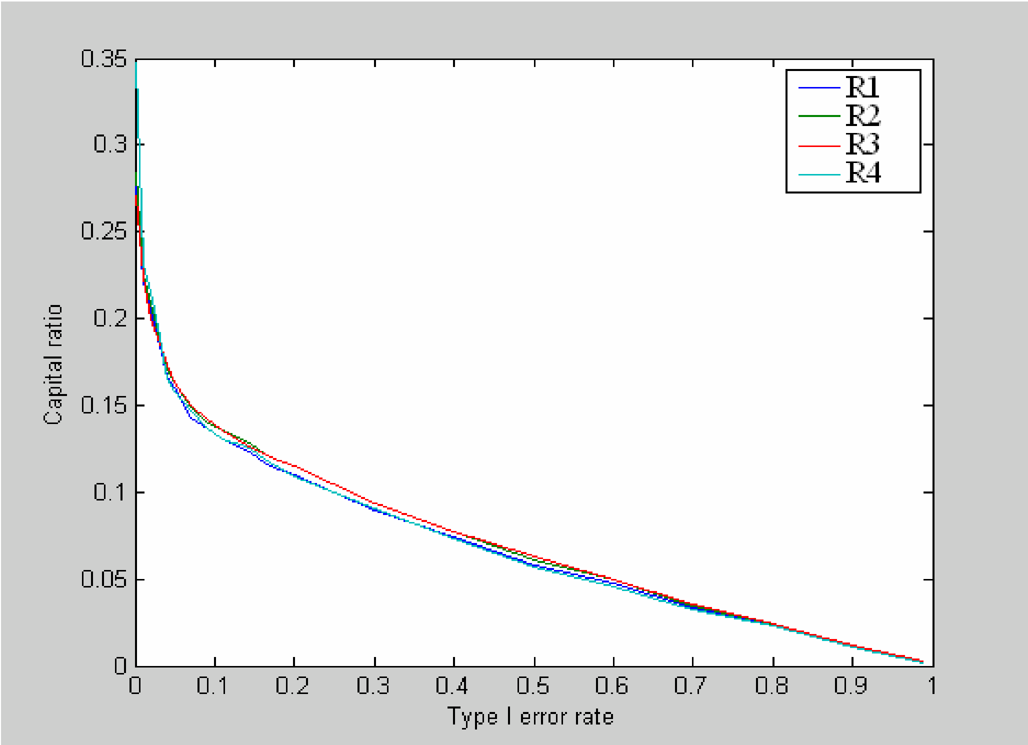


Figure 3: The minimal average capital ratios of alternative requirements to achieve target solvency probabilities

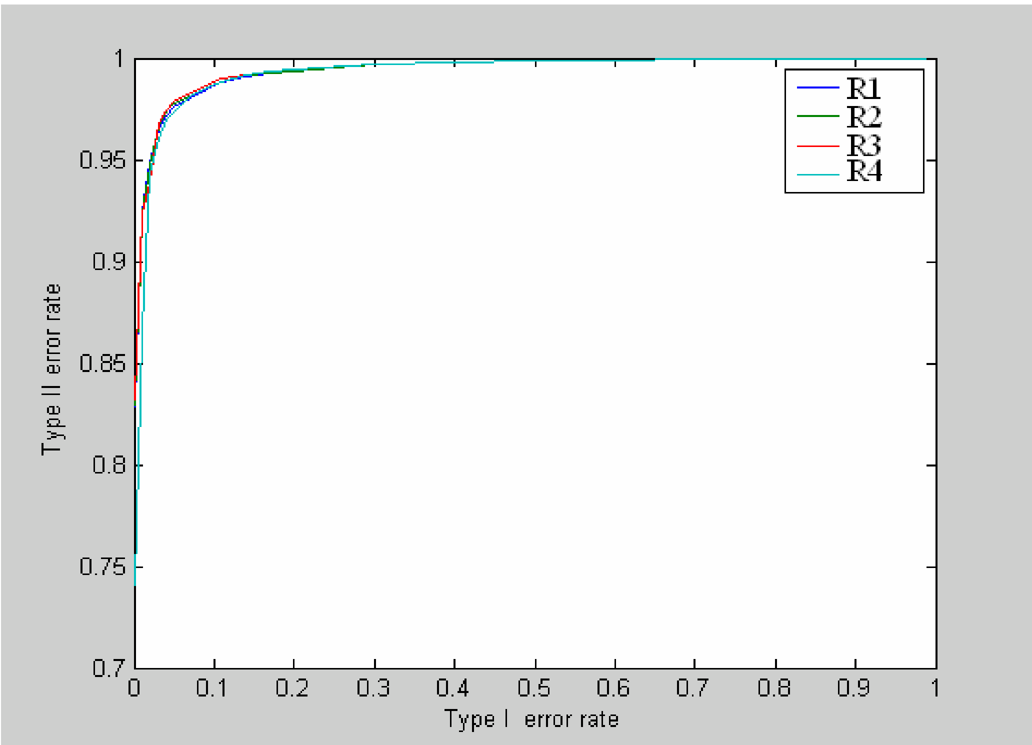


Figure 4: The ROC curves of alternative capital requirements within the hypothetical world of low correlation structure

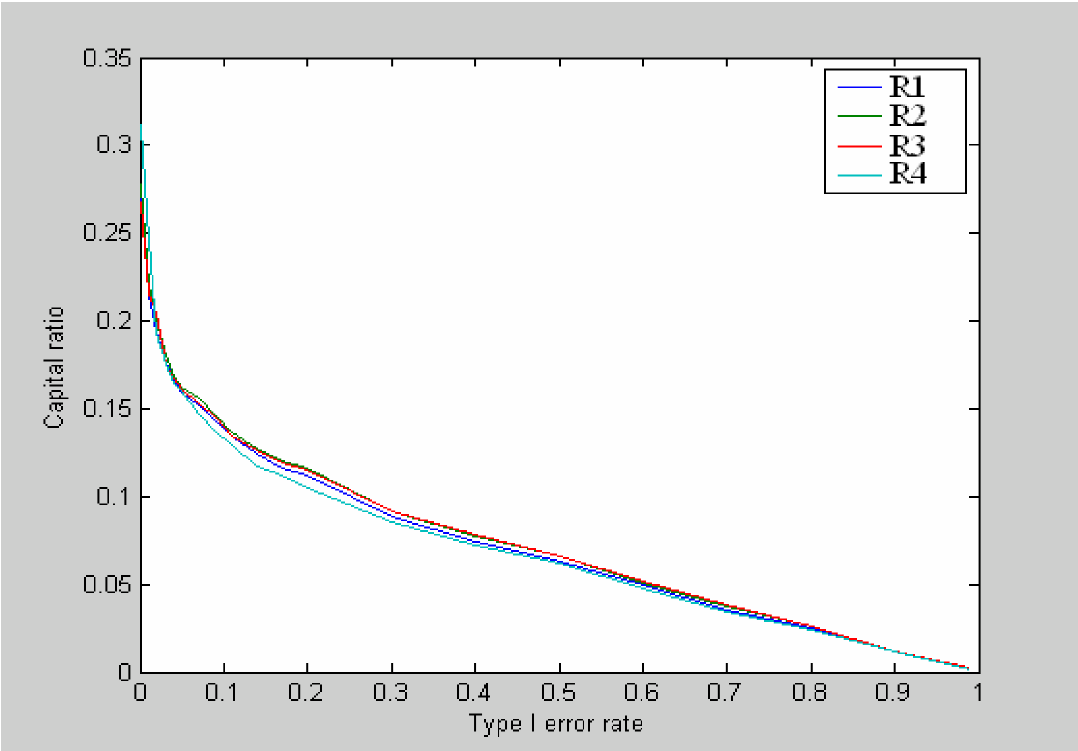


Figure 5: The capital ratio curves of alternative requirements within the hypothetical world of low correlation structure

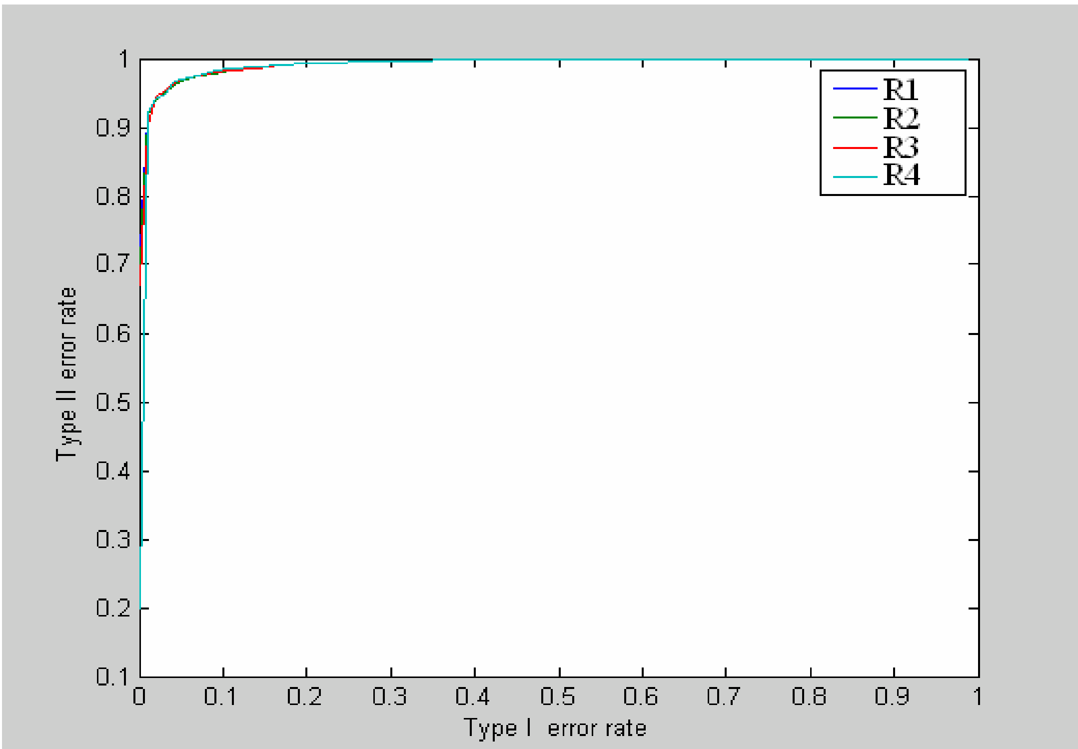


Figure 6: The ROC curves of alternative capital requirements within the hypothetical world of high correlation structure

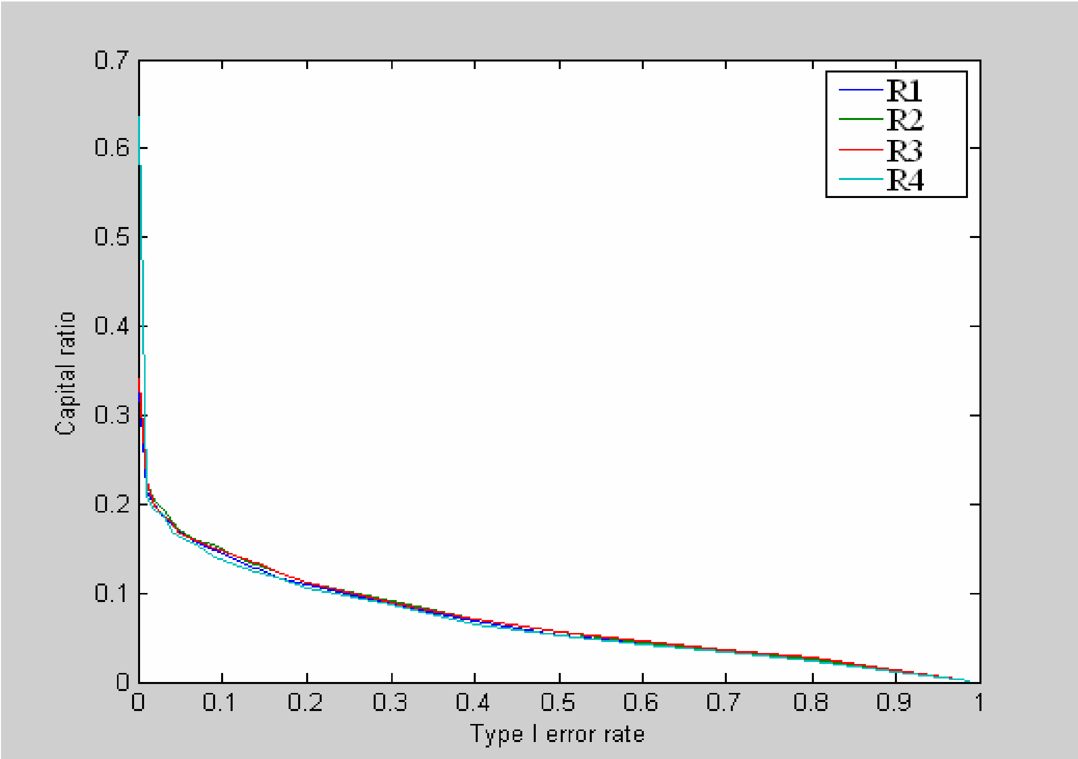


Figure 7: The capital ratio curves of alternative requirements within the hypothetical world of high correlation structure

Table 1: The type II error rates of capital requirements using alternative correlation specifications with selected type I error rates

Type I	0.00	0.01	0.02	0.03	0.04	0.05	0.07	0.09	0.11	0.14	0.17	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.99
R_1	0.1756	0.0824	0.0613	0.0399	0.0266	0.0228	0.0149	0.0125	0.0107	0.0092	0.0073	0.0063	0.0037	0.0024	0.0015	0.0011	0.0006	0.0004	0.0002	0.0000
R_2	0.1801	0.0801	0.0598	0.0382	0.0260	0.0212	0.0152	0.0120	0.0108	0.0092	0.0073	0.0064	0.0037	0.0024	0.0015	0.0011	0.0006	0.0004	0.0002	0.0000
R_3	0.1725	0.0881	0.0544	0.0382	0.0283	0.0219	0.0154	0.0125	0.0105	0.0085	0.0071	0.0062	0.0035	0.0023	0.0015	0.0010	0.0006	0.0003	0.0001	0.0000
R_4	0.3294	0.1115	0.0857	0.0530	0.0295	0.0245	0.0178	0.0137	0.0116	0.0101	0.0079	0.0064	0.0039	0.0024	0.0014	0.0010	0.0006	0.0004	0.0001	0.0000

Table 2: The minimal average capital ratios of alternative requirements to achieve selected target solvency probabilities

Target	1.00	0.99	0.98	0.97	0.96	0.95	0.93	0.91	0.89	0.86	0.83	0.80	0.70	0.60	0.50	0.40	0.30	0.20	0.10	0.01
R_1	0.2781	0.2219	0.2053	0.1843	0.1664	0.1601	0.1432	0.1363	0.1303	0.1242	0.1154	0.1103	0.0900	0.0737	0.0579	0.0475	0.0336	0.0230	0.0114	0.0020
R_2	0.2838	0.2252	0.2093	0.1881	0.1714	0.1628	0.1493	0.1401	0.1360	0.1291	0.1205	0.1152	0.0934	0.0771	0.0611	0.0495	0.0348	0.0241	0.0121	0.0021
R_3	0.2702	0.2249	0.2004	0.1854	0.1733	0.1630	0.1495	0.1417	0.1353	0.1271	0.1201	0.1151	0.0936	0.0770	0.0626	0.0494	0.0353	0.0242	0.0120	0.0021
R_4	0.3476	0.2320	0.2153	0.1899	0.1650	0.1580	0.1464	0.1365	0.1308	0.1259	0.1172	0.1093	0.0909	0.0736	0.0566	0.0457	0.0324	0.0233	0.0111	0.0020

Table 3: The type II error rates of alternative capital requirements within the hypothetical world of low correlation structure

Type I	0.00	0.01	0.02	0.03	0.04	0.05	0.07	0.09	0.11	0.14	0.17	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.99
R_1	0.1717	0.0759	0.0511	0.0358	0.0287	0.0238	0.0193	0.0153	0.0121	0.0090	0.0072	0.0065	0.0034	0.0022	0.0016	0.0010	0.0005	0.0003	0.0001	0.0000
R_2	0.1705	0.0767	0.0534	0.0342	0.0259	0.0222	0.0189	0.0144	0.0114	0.0086	0.0071	0.0063	0.0033	0.0022	0.0016	0.0010	0.0006	0.0003	0.0001	0.0000
R_3	0.1682	0.0769	0.0608	0.0320	0.0257	0.0216	0.0173	0.0140	0.0102	0.0082	0.0067	0.0059	0.0031	0.0022	0.0016	0.0010	0.0005	0.0003	0.0001	0.0000
R_4	0.2596	0.1349	0.0570	0.0395	0.0299	0.0272	0.0192	0.0142	0.0115	0.0080	0.0069	0.0057	0.0032	0.0022	0.0016	0.0010	0.0006	0.0003	0.0001	0.0000

Table 4: The minimal average capital ratios of alternative requirements within the hypothetical world of low correlation structure

Target	1.00	0.99	0.98	0.97	0.96	0.95	0.93	0.91	0.89	0.86	0.83	0.80	0.70	0.60	0.50	0.40	0.30	0.20	0.10	0.01
R_1	0.2743	0.2148	0.1936	0.1766	0.1673	0.1595	0.1515	0.1429	0.1343	0.1241	0.1164	0.1121	0.0884	0.0741	0.0634	0.0495	0.0358	0.0247	0.0115	0.0019
R_2	0.2776	0.2207	0.2014	0.1803	0.1686	0.1624	0.1563	0.1459	0.1371	0.1277	0.1205	0.1162	0.0919	0.0772	0.0664	0.0508	0.0376	0.0256	0.0120	0.0020
R_3	0.2671	0.2158	0.2044	0.1759	0.1672	0.1607	0.1527	0.1451	0.1341	0.1268	0.1197	0.1150	0.0912	0.0785	0.0665	0.0514	0.0383	0.0263	0.0120	0.0020
R_4	0.3118	0.2461	0.1927	0.1755	0.1640	0.1603	0.1473	0.1367	0.1293	0.1169	0.1119	0.1050	0.0857	0.0724	0.0621	0.0477	0.0345	0.0237	0.0114	0.0017

Table 5: The type II error rates of alternative capital requirements within the hypothetical world of high correlation structure

Type I	0.00	0.01	0.02	0.03	0.04	0.05	0.07	0.09	0.11	0.14	0.17	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.99
R_1	0.2814	0.0825	0.0617	0.0479	0.0411	0.0340	0.0256	0.0217	0.0176	0.0129	0.0095	0.0077	0.0041	0.0023	0.0013	0.0009	0.0006	0.0003	0.0001	0.0000
R_2	0.3013	0.0838	0.0606	0.0495	0.0398	0.0317	0.0243	0.0219	0.0171	0.0127	0.0094	0.0072	0.0040	0.0022	0.0013	0.0009	0.0006	0.0003	0.0001	0.0000
R_3	0.3301	0.0956	0.0566	0.0470	0.0393	0.0292	0.0237	0.0200	0.0170	0.0132	0.0091	0.0068	0.0036	0.0021	0.0012	0.0008	0.0005	0.0003	0.0001	0.0000
R_4	0.8010	0.0769	0.0585	0.0520	0.0358	0.0306	0.0251	0.0176	0.0153	0.0114	0.0091	0.0070	0.0041	0.0020	0.0013	0.0008	0.0006	0.0003	0.0001	0.0000

Table 6: The minimal average capital ratios of alternative requirements within the hypothetical world of high correlation structure

Target	1.00	0.99	0.98	0.97	0.96	0.95	0.93	0.91	0.89	0.86	0.83	0.80	0.70	0.60	0.50	0.40	0.30	0.20	0.10	0.01
R_1	0.3299	0.2169	0.1995	0.1857	0.1779	0.1688	0.1563	0.1492	0.1404	0.1278	0.1166	0.1090	0.0879	0.0693	0.0531	0.0431	0.0348	0.0244	0.0127	0.0005
R_2	0.3410	0.2232	0.2041	0.1931	0.1819	0.1710	0.1594	0.1549	0.1444	0.1324	0.1209	0.1113	0.0904	0.0715	0.0557	0.0447	0.0363	0.0258	0.0133	0.0005
R_3	0.3414	0.2263	0.1974	0.1882	0.1794	0.1668	0.1581	0.1510	0.1445	0.1348	0.1212	0.1110	0.0898	0.0709	0.0558	0.0456	0.0363	0.0267	0.0134	0.0006
R_4	0.6354	0.2080	0.1929	0.1869	0.1691	0.1623	0.1542	0.1399	0.1342	0.1235	0.1152	0.1061	0.0871	0.0648	0.0524	0.0412	0.0333	0.0236	0.0121	0.0005

Table 7: The type I and type II error rates of alternative capital requirements given various α

	α	2.326	2.054	1.645	1.282	1.036	0.842	0.675	0.524	0.385	0.253
R_1	Type I	0.0038	0.0113	0.0301	0.0847	0.1940	0.2938	0.4106	0.5160	0.6403	0.7514
	Type II	0.1248	0.0814	0.0340	0.0131	0.0066	0.0038	0.0024	0.0014	0.0009	0.0005
R_2	Type I	0.0038	0.0094	0.0264	0.0716	0.1676	0.2731	0.3917	0.5009	0.6290	0.7420
	Type II	0.1543	0.0999	0.0412	0.0152	0.0074	0.0041	0.0025	0.0015	0.0009	0.0005
R_3	Type I	0.0000	0.0000	0.0038	0.0188	0.0527	0.1243	0.2411	0.3729	0.5085	0.6686
	Type II	0.3760	0.2894	0.1447	0.0544	0.0214	0.0096	0.0048	0.0027	0.0014	0.0007
R_4	Type I	0.0038	0.0151	0.0320	0.0829	0.1827	0.2863	0.3955	0.4972	0.6384	0.7458
	Type II	0.1530	0.0996	0.0426	0.0151	0.0072	0.0040	0.0024	0.0014	0.0009	0.0005