

On the Economics of Life Settlements

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Abstract

Life settlements provide the possibility for individuals to convert their life insurance policies into cash in a secondary market transaction. Hence, this additional option is welcome from the current policyholders' point of view, but the reasons for the attractiveness to financial investors as well as the equilibrium impacts on life insurers and future policyholders are not immediate clear.

This paper presents economic models to analyze and explain the details of life settlement transactions. Moreover, we investigate the potential welfare effects of a secondary life insurance market. One of our key findings is that although a perfect market for life settlements could be welfare-enhancing, high fixed costs prevailing in the transactions may alleviate or even invert this effect.

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1 Introduction

Life settlements have become increasingly important in the life insurance industry (Hodson (2009)). Within such a transaction, both the liability of future contingent premiums and the benefit of the life insurance contract are transferred to a life settlement company in return for which the former policyholder receives a single lump-sum payment. While initially the target market primarily consisted of policyholders with lethal diseases (e.g., terminal cancer or AIDS) or very short life expectancies (so-called viatical settlements), in recent years life settlements have become attractive for a broader array of policyholders.

There are many reasons why a policyholder may no longer want to keep her insurance contract. For example, the insured's need for coverage may decline over time or the insured may face a pressing need for liquidity. The traditional options in such a scenario are to either surrender the policy (respectively to let it lapse if the surrender value is non-positive) or to discontinue premium payments. The existence of life settlements provides policyholders with a third option. Due to competition (or even as required by regulation in some U.S. states), the purchase price of a life settlement will always be higher than the associated surrender value. Therefore, without considering the cost associated with settling, given that a policyholder is eligible to settle, she will always prefer settling to surrendering. However, the life settlement could be regarded as a "potential threat" to the insurance company since premium calculations are usually based on historic surrender rates and surrender profits.

Several authors have investigated the growing market for life settlements although most have directed their attention towards investors or industry professionals (see e.g. Modu (2004)) or they have addressed issues related to the securitization of life settlements (see e.g. Stone and Zissu (2006)). To date, few papers have investigated the economic effects of a secondary market for life insurance policies.

One exception is the study by the consulting firm Deloitte (2005), which conducted an actuarial valuation of life settlements to compare typical life settlement values with the intrinsic economic value for policyholders with impaired health. The authors conclude that policyholders maximize their estate value by liquidating other assets and keeping their life insurance policies in force. This, in turn, would imply that the welfare effects of a secondary market for life insurance are minor.

Doherty and Singer (2003) claim even though some increase in the cost of insurance may be expected, this increase does not necessarily imply a negative effect on consumer welfare. Instead, the authors argue that the introduction of a secondary market for life insurance policies reduces the monopsony power of insurers and, hence, will lead to an improvement of welfare for both existing and future policyholders.

In a recent contribution, Daily et al. (2008) challenge the Doherty and Singer (2003) conclusion.

They argue that in a regime with a secondary life insurance market, front-loading of insurance contracts leads to a disadvantageous distribution of costs over the policyholder's lifetime. More precisely, life settlements shift resources from the initial stages of the working period of a person's life to the settling events, and since income is typically lower early in life and because resources are partially wasted if the insured decides to drop coverage, the consequences on welfare are potentially negative.

However, their discussion ignores that consumers' utility may depend on their health states. Moreover, important contractual features of life settlements are neglected. The current paper addresses these issues: We propose a two-stage model with possible asymmetric information as well as a three-stage model to consider the effects of the secondary life market on current and future policyholders, respectively. Our results reveal that even though a perfect secondary life market could be welfare-enhancing, high transaction costs as typically observed within life settlement transactions may alleviate or even invert this effect.

The remainder of this paper is organized as follows. After briefly discussing the impact of a secondary market for life insurance policies from various points of view and providing a concise description of a typical life settlement transaction in Section 2, Section 3 develops a two-stage model for such transactions as a dynamic game between the life settlement company and the policyholder. In particular, we discuss the cases with and without asymmetric information with regards to the policyholder's health state in Section 3.1 and 3.2, respectively. Subsequently, we address the question how a life insurance company will react to the emergence of a secondary life market by adjusting the price of insurance in a three-stage model. Similarly to Doherty and Singer (2003), we find that the emergence of a secondary market generally results in an increase of prices due to the reduced surrender rate of policyholders in both health states. We analyze the impact of this price adjustment on future policyholders with different characteristics, and particularly study how the total social welfare is affected by the introduction of a secondary life market. We conclude with a brief summary of our main results and a discussion of limitations of our approach as well as potential future research directions.

2 The Market For Life Settlements

Traditionally, the only option for the owner of a life insurance contract to convert her policy into cash was to surrender it to the life insurer as the monopsonist at some predetermined value, the so-called (cash) surrender value. In contrast, within a life settlement transaction, the (secondary) market price is determined based on the assessment of the individual policy (Frank (2005)). In particular, the policy will only be settled if the settlement value exceeds the surrender value, so that (current) policyholders can only benefit from this additional option.

For ultimate investors, on the other hand, assets backed by 'used' life insurance policies provide investment opportunities that are likely to have relatively low correlation with market systematic risk, making them valuable for diversification purposes (Cowley and Cummins (2005)). Moreover, frequent transactions of life insurance policies between their holders and life settlement companies may induce a healthier and more complete insurance market. In particular, an active secondary market may lead to the erosion of the insurers' ability to extract monopsony rents from early policy terminations (Doherty and Singer (2003)).

Hence, life settlements could benefit all existing policyholders, investors and future policyholders. Doherty and Singer (2003) argue that even insurance companies may benefit from a secondary market by an increasing demand for insurance due to the increased liquidity. However, there are also potentially negative repercussions on future policyholders and insurance companies due to front-loading and the erosion of surrender profits, respectively, as also pointed out in the introduction.

In order to analyze and compare these different effects, it is important to include all potentially relevant properties and details of a typical life settlement transaction in our model. The transaction process evolves as follows:

First, a life settlement broker finds directly or among insurance agents cases in which policyholders want to evaluate their options of cashing out their life insurance policies. The broker then will obtain offers from life settlement companies and policyholders will decide whether or not to settle their policies based on the provided prices, while necessary health examinations are required by life settlement companies. Life settlement companies will find investors to purchase policies and pay contingent future premiums possibly by the process of securitization. A commission is paid to the broker, and a lump sum amount is paid to the former policyholder to transfer the ownership and right to the insurance benefits to the investors.

There is no insurable interest after the life settlement contract is assigned. Therefore, there is the potential risk of adverse selection for the life settlement companies or more precisely, the investors. If we assume that there is no mistake in the health examination, then no information asymmetries arise. Hence, the problem reduces to the risk associated with the accuracy of the estimation of the individual life table applied in the settlement company's calculation and other systematic risks such as interest rate risk or inflation risk. However, the possibility of an incorrect diagnosis is inevitable. For example, if a relatively healthy policyholder is incorrectly diagnosed as sick, she will have an incentive to pretend being sick and cash out her insurance policy.

Our objective in this paper is to assess the impact of a secondary market on current and future policyholders. Therefore, we disregard a potentially fraudulent behavior of brokers by assuming that they behave according to the policyholder's interests and by neglecting the commission. Similarly, we disregard tensions between the life settlement company and the investors. Hence, in our

simplified model, for existing life insurance policies, there are only two parties involved in the transaction: the policyholder and the life settlement company. However, for future policies, we also need to incorporate the life insurer to reflect the price adjustment of insurance policies.

3 Life Settlements on Existing Policies

In this section, we investigate how life settlements will affect existing life insurance policies. For these policies, their contract details such as premiums or death benefits have already been specified at the time the policies were signed and life insurers have no possibility to revise these conditions as a reaction to the emergence of life settlements. Therefore, we only need to consider the positions of a representative risk-averse policyholder and a risk-neutral life settlement company as well as their interaction, which can be formalized as a dynamic game in a two-stage model: In the first stage, the life settlement company provides the purchasing price based on given characteristics and then the policyholder makes her contingent decision in order to maximize her welfare in the following stage. In particular, initially we assume that all relevant information concerning the policyholder can be assessed by the life settlement company, so it will choose the optimal set of purchasing prices to maximize its expected profit.

However, similarly to other applications of contract theory, information asymmetry may play an important role here. More specifically, asymmetric information could originate from the real health state of the policyholder not being correctly observed by the life settlement company. Hence, after considering the basic case without asymmetric information in 3.1, we develop a model with non-perfectly observed health states in the second part of this section.

3.1 Existing Policies: A Two-stage Model without Asymmetric Information

The case without asymmetric information is the simplest case and is also discussed in other contributions (see e.g. Daily et al. (2008)). However, none of them treat settling as endogenous: A probability that the insurance policy is no longer 'needed' is assumed (e.g., ceased bequest motive due to divorce or widowling) and this exogenous shock is the only driving force for settling or surrendering. Here, we additionally consider an endogenous reason: Policyholders could choose to settle their policies simply due to welfare maximization even if the policy itself is still technically 'needed'. We incorporate the exogenous case by adding a shock into the bequest function. Therefore, our results naturally nest the Daily et al. (2008) case.

For now, assume that there are three health states: active (a), sick (s) and dead (d) with a different given utility function in each state. A second variable with value in $\{0, 1, 2\}$ describes the state of the policy: '0' indicates that the policyholder keeps her policy at the beginning of the current period

(for example, the current calendar year) and is still paying premiums; '1' means the policyholder has already settled her policy to the life settlement company; and '2' denotes that the policy has been surrendered. For the state corresponding to the exogenous need for insurance, 'n' denotes that the shock to the bequest function has not yet happened, while 'u' indicates that the shock has already applied, that is, the policy is no longer 'needed' by the policyholder.

An exemplary value function can be written as:

$$V_{x,x_0}(W, a, 0, n) = \max \{ \mathbf{1}_{\{W \geq \alpha(x_0)F\}} V_{x,x_0}^0(W, a, 0, n), V_{x,x_0}^1(W, a, 0, n), V_{x,x_0}^2(W, a, 0, n) \}, \quad (1)$$

in which $V_{x,x_0}^0(W, a, 0, n)$, $V_{x,x_0}^1(W, a, 0, n)$, $V_{x,x_0}^2(W, a, 0, n)$ denote the value function if the policyholder chooses to retain, settle and surrender her policy, respectively, at age x with beginning-of-period wealth W and beginning-of-period state $(a, 0, n)$. Moreover, F is the face value of the policy and $\alpha(x_0)F$ denotes the annual premium where x_0 is the age when the policy was purchased. According to the utility maximization paradigm, the policyholder will choose the option with the highest value function. Notice that by the dynamic programming principle, each value function can be written as a recursive Bellman equation:

$$\begin{aligned} V_{x,x_0}^i(W, a, 0, n) = & \max_c \{ U(c) + \beta \times [\gamma p_{aa}^x V_{x+1,x_0}(W', a, i, n) + \gamma p_{as}^x V_{x+1,x_0}(W', s, i, n) \\ & + \gamma(1 - p_{aa}^x - p_{as}^x) V_{x+1,x_0}(W', d, i, n) + (1 - \gamma) p_{aa}^x V_{x+1,x_0}(W', a, i, u) \\ & + (1 - \gamma) p_{as}^x V_{x+1,x_0}(W', s, i, u) \\ & + (1 - \gamma)(1 - p_{aa}^x - p_{as}^x) V_{x+1,x_0}(W', d, i, u)] \}, \quad i = 0, 1, 2. \end{aligned} \quad (2)$$

with varying constraints for different values of i . More precisely, if $i = 0$, the constraint will be:

$$\begin{aligned} W' &= (W - c - \alpha(x_0)F) \times (1 + r), \\ 0 &\leq c \leq W - \alpha(x_0)F, \end{aligned}$$

where c is the current period's consumption, β is the time discount rate between periods, γ denotes the probability for a shock to the bequest function, p^x as the transition probability between different health states, and r as the risk-free interest rate. If $i = 1$, on the other hand, we obtain:

$$\begin{aligned} W' &= (W - c - K + \rho(a, x, x_0) \times F) \times (1 + r), \\ 0 &\leq c \leq W + \rho(a, x, x_0) \times F - K, \end{aligned}$$

where K is the cost associated with the mandatory health examination, i.e. the cost associated with settling which we assume is independent of age and the health state, and $\rho(a, x, x_0)$ is the purchasing rate provided by the life settlement company, which is a function of the health state,

current age and the age of purchasing. Finally, if $i = 2$, the constraints becomes:

$$\begin{aligned} W' &= (W - c + \zeta(x, x_0) \times F) \times (1 + r), \\ 0 &\leq c \leq W + \zeta(x, x_0) \times F, \end{aligned}$$

where $\zeta(x, x_0)$ is the surrender rate from the life insurance company and, thus, is independent of the individual's health state.

The remaining equations and necessary boundary conditions are provided in Appendix A. Based on these equations, together with utility functions with desired properties, the backward induction method can be applied to solve for the policyholder's optimal decision. Notice that when β is sufficiently small, there is always positive probability that the policyholder will settle her policy due to the endogenous economic reason.

Denote by $\rho(h, x, x_0)$ the purchasing rates prescribed by the life settlement company in stage 1, in which $h \in \{a, s\}$ stands for different health states. Further, define λ as the measurement of risk aversion of the policyholder. Therefore, the policyholder's insurance relevant strategy for given ρ, W, F and λ , $\phi_{\rho, W, F, \lambda}(\tau)$, is a random variable contingent on the path of the policyholder's health states, τ , with associated probability $P(\tau)$, which can be obtained by a life table.

For any given realization τ , if the policyholder eventually chooses to settle her policy before she dies, denote $(X(\tau), H(\tau))$ as the age and health state of the policyholder at the time of settling, then the life settlement company's profit is:

$$\pi(\tau) = \frac{1}{(1+r)^{X(\tau)-x}} [PV(H(\tau), X(\tau), x_0) - \rho(H(\tau), X(\tau), x_0) \times F)], \quad (3)$$

where x denotes the current age of the policyholder, $PV(H(\tau), X(\tau), x_0)$ is the present value of the insurance policy at age $X(\tau)$ and health state $H(\tau)$. On the other hand, if the policy is being kept until the policyholder dies, the profit of the life settlement company is 0. Notice that π is also a random variable contingent on τ ; therefore, the expected profit the life settlement company could gain from the policyholder is:

$$\Pi = E(\pi) = \sum_{\tau} P(\tau) \times \pi(\tau). \quad (4)$$

Π is a new function of ρ, W, F and λ . If we denote the density of x, x_0 and health state h of the targeted population of policyholders by $f(h, x, x_0)$, the distribution of F, W, λ as $g(W, F, \lambda)$, then the life settlement company will maximize its total expected profit by choosing optimal set of $\rho(h, x, x_0)$ based on the following function:

$$\max_{\rho} \int \int \int \int f(h, x, x_0) g(W, F, \lambda) \Pi(\rho(h, x, x_0), W, F, \lambda) dh dx dx_0 dF dW_0 d\lambda. \quad (5)$$

There are several implicit assumptions in the above two-stage model. Life settlement companies are mostly interested in policies of senior policyholders, so we assume that the representative policyholder is retired with no period income. The restriction can be relaxed by allowing the policyholder to receive period income such as a pension. We further assume that the life settlement company has the monopoly power and could achieve positive profit; however, as illustrated in the following subsection, we could also assume competition among life settlement companies by changing their objection functions. Moreover, more insurance choices could be added for the policyholder such as allowing her to partial surrender the policy, to lapse the policy, or to buy additional insurance.

3.2 Existing Policies: A Modified Model with Asymmetric Information

Information asymmetry and, consequently, adverse selection and moral hazard are potentially predominant in insurance transactions (see e.g., Puelz and Snow (1994), Chiappori and Salanie (2000) or Chiappori (2000) for an overview), although Cawley and Phillipson (1999) do not find evidence for asymmetric information in life insurance. In a recent paper, Biffis and Blake (2009) investigate the effect of asymmetric information on the securitization of life insurance products. In their model, there is information on the policyholders known by the life insurer but not observable by the outside investors, and the life insurer uses the percentage of self-retained policies as a credible signal. In contrast, here we consider potential information asymmetries between a policyholder and a life settlement company with a different signal mechanism.

Assume that the policyholder always knows her true health state while the life settlement company can only make an inference with the accuracy affected by the volume of medical documents provided by the policyholder. The more documents are provided, the more accurate the estimation becomes and the higher is the associated cost the policyholder needs to bear. Further assume the volume of medical documents is also observed by the life settlement company, so it does not only affect the precision of the estimation, but also serves as a credible signal from the policyholder.

To accommodate these differences, some modifications are put on the previous model. Health states are assumed to be distributed continuously, and to avoid unnecessary complexity, we assume that there are only two time periods and the insurance policy is a one-period term insurance. The policyholder is endowed with this policy, an initial wealth and a health state at the beginning of the first period; the policy expires the end of the first period and all policyholders will die for sure by the end of the second period. While the latter simplifications should not affect the qualitative results of the model, the assumption of continuous health states allows us to focus on potential

implications of asymmetric information.

In order to give some explicit analysis, we additionally assume that the secondary life insurance market is competitive. In particular, every life settlement company will eventually obtain zero expected total profit, that is, for each single policy purchased, its expected profit should be zero:

$$E(\pi(\bar{\mu}, \eta, F)) = E[((1 - e^{-\mu}) \times F - (1 + r) \times OP(\bar{\mu}, \eta, F)) \mathbf{1}_{\{OP(\bar{\mu}, \eta, F) \geq \xi(\mu, F)\}}] = 0, \quad (6)$$

where $\mu > 0$ is the force of mortality of the policyholder over period 1 and different μ correspond to different health states. Note that, $1 - e^{-\mu}$ is the one period mortality rate. By η we denote the cost of medical documents chosen by the policyholder, which is chosen optimally before obtaining a quote from the life settlement company and is observable by the life settlement company. $\bar{\mu}$ is the life settlement company's estimation for the force of mortality, which is assumed to follow a Gamma distribution for given μ and η : $\bar{\mu} \sim \text{Gamma}(\mu\eta, \frac{1}{\eta})$. Therefore, the estimation is unbiased and the larger η , the smaller is the estimation error (variance). $OP(\bar{\mu}, \eta, F)$ is the offer price given by the life settlement company based on estimated $\bar{\mu}$ and observed η . $\xi(\mu, F)$ is the reservation price for the policyholder, i.e. the policyholder will only settle her policy given that the offer price is greater than or equal to her reservation price.

If $f(\cdot)$ and $g(\cdot)$ denote the unconditional probability densities of μ and $\bar{\mu}$, respectively, the Bayesian rule can be applied to get the conditional distribution of μ based on $\bar{\mu}$ and η . Conversely, when supposing certain posterior distributions, a suitable prior distribution could also be determined in a similar fashion. While this approach may result in unusual assumptions on the unconditional distribution, it allows for a convenient form of the posterior distributions, so we accept this potential short coming. More specifically, we assume that the conditional distribution of μ given $\bar{\mu}$ is also Gamma with parameters $(\bar{\mu}\eta, \frac{1}{\eta})$. Finally, $\xi(\mu, F)$ is assumed to be a known function independent of η ¹ and increasing in μ .² Now, let $\Lambda(\cdot, F)$ denote the inverse function of $\xi(\cdot, F)$, therefore, $OP(\bar{\mu}, \eta, F) \geq \xi(\mu, F)$ is equivalent to $\mu \leq \Lambda(OP(\bar{\mu}, \eta, F), F)$ and we could write the expected

¹ η is chosen prior to obtaining the offer price from the life settlement company and can be seen as the 'sunk cost'.

²This is because the larger μ is, the less the survival probability will be, and thus the higher value of the term insurance policy.

profit as:

$$\begin{aligned}
& \mathbb{E}[(1 - e^{-\mu}) \times F - (1 + r) \times OP(\bar{\mu}, \eta, F)] \mathbf{1}_{OP(\bar{\mu}, \eta, F) \geq \xi(\mu, F)} \\
= & \mathbb{E}[(1 - e^{-\mu}) \times F - (1 + r) \times OP(\bar{\mu}, \eta, F)] \mathbf{1}_{\mu \leq \Lambda(OP(\bar{\mu}, \eta, F), F)} \\
= & \int_0^{\Lambda(OP(\bar{\mu}, \eta, F), F)} ((1 - e^{-\mu}) \times F - (1 + r) \times OP(\bar{\mu}, \eta, F)) f(\mu | \bar{\mu}, \eta) d\mu \\
= & (F - (1 + r)OP(\bar{\mu}, \eta, F)) \times Gamcdf(\Lambda(OP(\bar{\mu}, \eta, F), F), \bar{\mu}\eta, \frac{1}{\eta}) \\
& - F(\frac{\eta}{\eta - 1})^{\bar{\mu}\eta} \times Gamcdf(\Lambda(OP(\bar{\mu}, \eta, F), F), \bar{\mu}\eta, \frac{1}{\eta - 1})), \tag{7}
\end{aligned}$$

in which $f(\mu | \bar{\mu}, \eta)$ is the conditional distribution of μ and $Gamcdf(\cdot, a, b)$ is the cumulative distribution function of a Gamma distribution with parameters a and b . Therefore, for given $\bar{\mu}$, η and F , the only unknown variable in the expected profit above is the offer price $OP(\bar{\mu}, \eta, F)$, which could then be solved by letting the expected profit equal to zero.

Notice that Equation (7) resembles the well-known Black-Scholes formula for pricing European Put and Call options. This similarity is not a coincidence: In the case of the financial derivative, the holder will choose to exercise the option or not depending on whether exercising implies positive value. For example, in case of a Call option, the holder will only exercise if the underlying stock price exceeds the strike price. Similarly, for the life insurance policyholder, after obtaining the offer price from the life settlement company, she will also choose to settle her life insurance policy only if the offer price exceeds her reservation price. Therefore, the integration is limited with an upper bound and we obtain a similar equation. However, notice that the Black-Scholes formula directly gives the value of an option, while here we evaluate the offer price implicitly by setting the equation equal to zero.

For the policyholder, aside from consumption and insurance related choices similar to the symmetric information case, in additional it is necessary to choose η at the beginning of the first period:

$$\eta(\mu, F) = \arg \max \mathbb{E}[V^1 \times \mathbf{1}_{\{OP(\bar{\mu}, \eta, F) \geq \xi(\mu, F)\}} + V^2 \times \mathbf{1}_{\{OP(\bar{\mu}, \eta, F) < \xi(\mu, F)\}}], \tag{8}$$

where V^1 and V^2 are the value function of the policyholder in case she chooses to settle her policy or not, respectively, and they could be defined similarly as in the symmetric information case. Given $OP(\cdot, \cdot, F)$, V^1 and V^2 , the policyholder will be able to choose the optimal η to maximize her expected welfare. The remaining decisions (consumption choice, etc.) are similar to the symmetric information case, so we omit the detailed discussion and turn to implications of asymmetric information.

First, asymmetric information may be able to explain the puzzle of seemingly high excess returns in

the real life settlement investment market. On one hand, to the life settlement company or eventual investors, the profitability of a life settlement contract is primarily based on the policyholder's mortality rate, which should be close to independent from the capital market and thus has a "Beta" close to zero. According to the Capital Asset Pricing Model, it should hence have an almost zero excess return. On the other hand, when life settlement companies sell purchased policies in the capital market, the policy illustrations usually promise a high rate of return. A potential explanation is the incomplete competition or coalition among life settlement companies, yielding a real "Alpha". Another explanation, however, is that the high excess rate of return promoted by life settlement companies is calculated in an ambiguous way. Notice that the policyholder will only settle her policy if the offer price exceeds the reservation price. The asymmetry of the settling choice causes the asymmetry of the pricing rule. More specifically, even if the conditional mean of $\bar{\mu}$ is μ and the life settlement company is risk neutral, it will not directly use $\bar{\mu}$ to price the policy because the policyholder will reject the offer when $\bar{\mu}$ is far less than μ , that is, the value of the policy is far underestimated. However, she will be happy to take the offer when $\bar{\mu}$ exceeds μ . Since the life settlement company can not observe μ , it is not able to determine if the offer price is too low or too high before the policyholder makes her choice. Hence, as a way to balance and obtain zero expected profit in the competitive equilibrium, the life settlement company has to shift its pricing schedule to cover the possible tail loss by subtracting a certain amount from prices calculated by $\bar{\mu}$. These reduced offer prices, while not implying a real profit, may lead to the illusion of high excess returns. To sum up, although the unconditional expected profit based on the offer price is positive, the real profit conditioning on the acceptance of the policyholder may be zero.

To illustrate differences between the symmetric and the asymmetric information case, a simplified example is used here to compare the difference between the two offer prices based on the assumptions made in this section. In the case of symmetric information, together with the assumption of a competitive secondary life insurance market, the life settlement company will pay $\frac{(1-e^{-\mu})F}{(1+r)}$ in exchange of the policyholder's one-period term insurance. In the case of asymmetric information, on the other hand, $OP(\bar{\mu}, \eta, F)$ is derived by Equation (7). Assume that $\xi(\mu, F) = \frac{(1-e^{-\mu})F}{(1+r)}$, then $\Lambda(OP(\bar{\mu}, \eta, F), F) = -\ln(1 - \frac{OP(\bar{\mu}, \eta, F)(1+r)}{F})$, so we can obtain the numerical solution of $OP(\bar{\mu}, \eta, F)$.

In Figure 1 and 2, offer prices for the symmetric and the asymmetric information case, respectively, are displayed as a function of μ and F . η is fixed at 50 and the interest rate $r = 0.05$. Moreover, in Figure 3 the differences between the prices in the two cases are plotted. We first observe that the entire surface is below zero, which indicates the shift of the life settlement company's pricing schedule and could be used to explain the seemingly high excess rates of return. The difference approaches zero when $\bar{\mu}$ approaches 10, which signifies that 10 could be regarded as one extreme case, i.e. a trustworthy representation of infinity and there is little room left for adverse selection

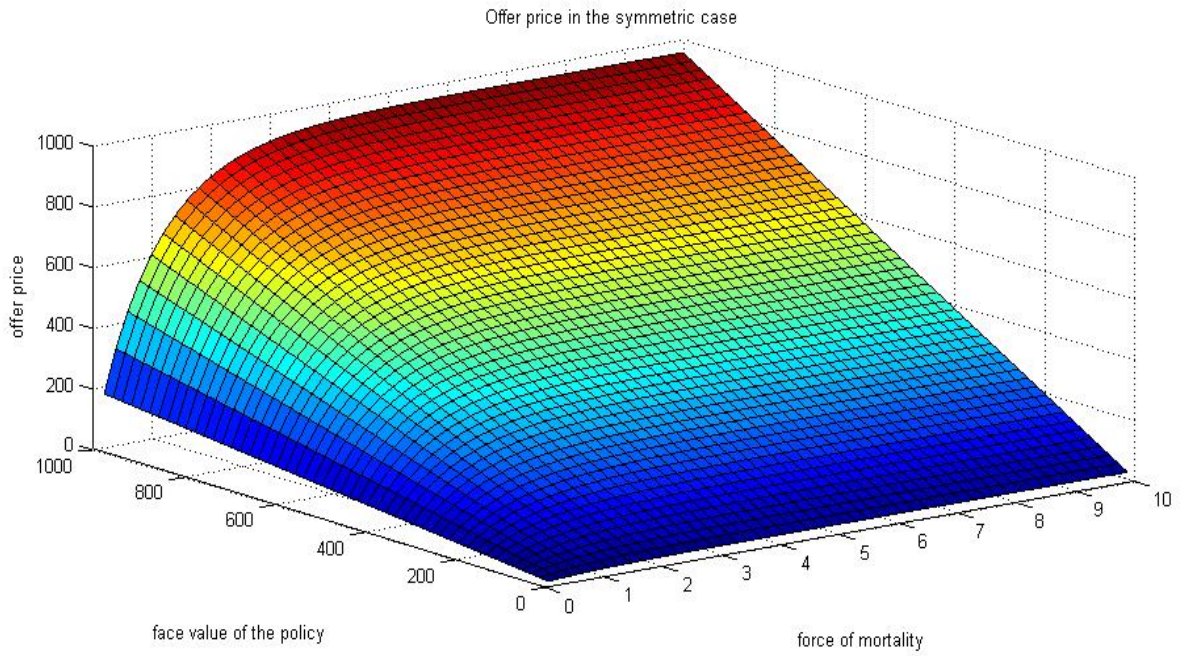


Figure 1: Symmetric case

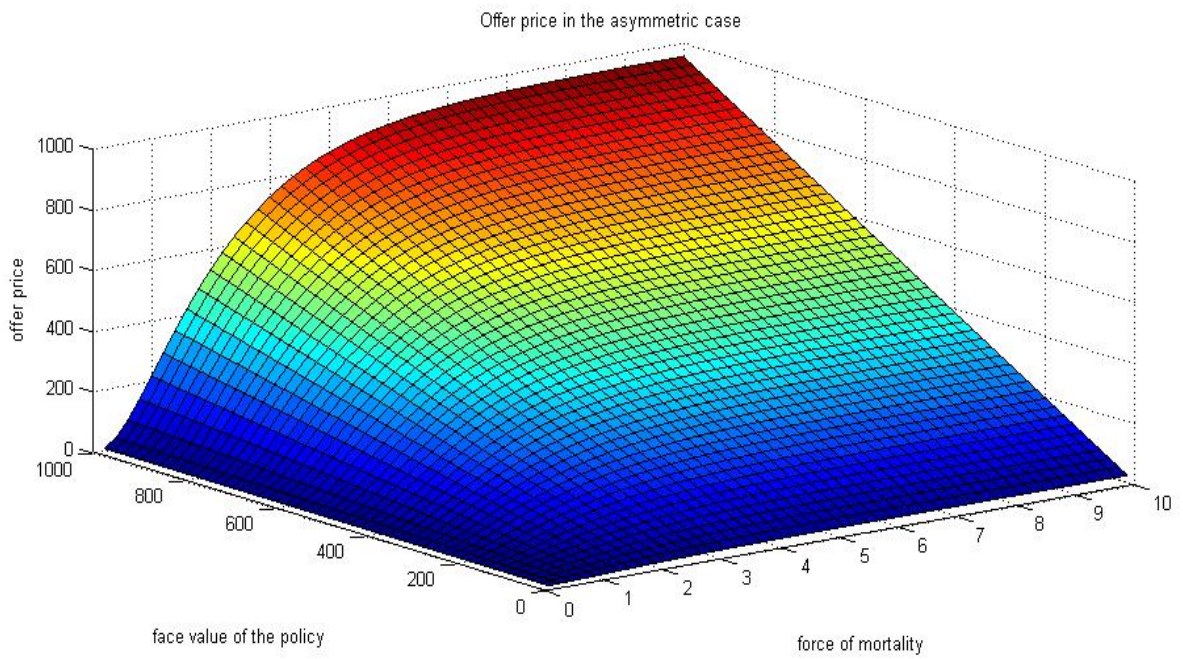


Figure 2: Asymmetric case

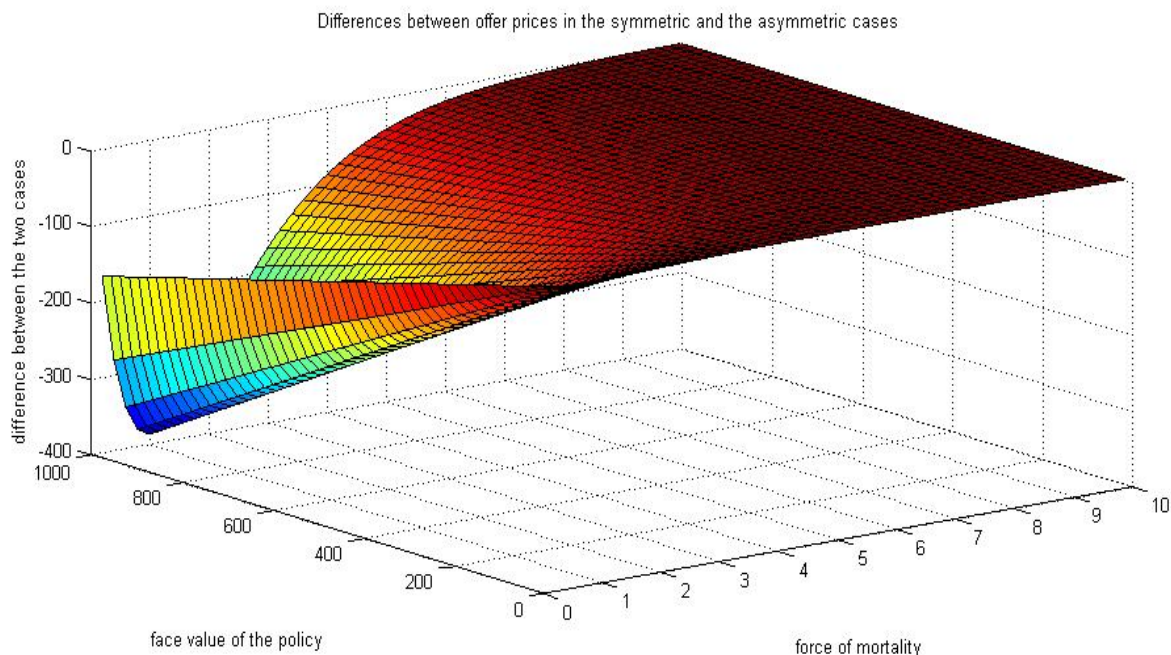


Figure 3: Differences

from the policyholders at this point. The other extreme case is when $\bar{\mu}$ approaches 0. Even if not shown directly in Figure 3, we can infer from Figure 2 that when $\bar{\mu}$ approaches 0, the offer price in the asymmetric case will approach 0, while in the symmetric case, it can be directly calculated as 0. Therefore, the difference also approaches zero as $\bar{\mu}$ approaches 0, but with a difference explanation: It is not because of the absence of possible adverse selection, but just for the reason that the offer price in the asymmetric case should always be non-negative and less than the one in the symmetric case, which approaches to 0 as $\bar{\mu}$ approaches 0.

The parameter η also plays an important role. Generally, policyholders with impaired health state would tend to choose a large η and policyholders holding policies with large face values would tend to choose a large η . The reason is that their marginal utility of settling is higher in this case and η is relatively small compared with the offer price. In order to guarantee some minimum level of accuracy in the estimation of force of mortality μ to reduce the severity of potential adverse selection, usually a minimal amount of medical documentation is required by life settlement companies, which may preclude relatively healthy and poor policyholders from settling and may thus affect the social welfare. We will continue the analysis of the impact of a secondary market on social welfare in latter sections.

4 Life Settlements on Future Policies

In the previous sections, we focus on policies that have already been sold to policyholders, i.e. before the emergence of life settlements. Therefore, their premiums have been set and cannot be adjusted by life insurers. In particular, there was no necessity to consider life insurers' actions in these sections. However, just as life settlements may change the behavior of policyholders such as their surrendering choices, it is inevitable that life insurers will adjust premiums accordingly when designing future policies. Therefore, to analyze the long term welfare effects of a secondary life insurance market, it is necessary to incorporate the life insurer as a third party, which is the objective of this section. We begin by introducing our underlying model assumptions, followed by a discussion of the policyholder's, the life settlement company's and the life insurer's perspectives, respectively.

4.1 Model Assumptions

We propose a three-stage model to reflect the relationships among a life insurer, a life settlement company and a policyholder with a similar logic as in section 3. In the first stage, the life insurer provides the price of the life insurance policy; in the second stage, the life settlement company determines the settlement price; while, in the third stage, the policyholder chooses her period consumptions and makes the decisions regarding the insurance contract.

For the sake of parsimony and in view of the ultimate goal of assessing welfare implications, we assume complete competition in both primary and secondary life insurance markets and no asymmetric information. Moreover, similar to section 3.2, we only consider three periods to roughly represent a life-cycle of the policyholder: The first period is when individuals are young and at work, while the potential loss of family income induces the demand of insurance; at the beginning of the second period, agents retire with no further income; in particular, they may choose to surrender or settle their policies to attain higher utilities; in order to distinguish different health states, a third period is used corresponding to the extremely senior age period – while healthy and sick policyholders have different probabilities of reaching this period, both types will certainly die at the end of it. The actual durations of the three periods may differ. For example, the working period may be significantly longer than the extreme senior period and this difference may affect some factors such as the payment of premiums or discount rates. We use n_1, n_2, n_3 to denote the actual length of each period.

We consider only whole-life insurance contracts here. Assume there are three health states: active (a), sick (s) and dead (d) with state dependent utility functions: $U(\cdot)$, $\tilde{U}(\cdot)$ and $B(\cdot)$, respectively. β is the time discount rate. Policies are sold at the beginning of the first period and we assume all policyholders are in state a at that time endowed with initial wealth y distributed according

the density $f(y)$ across the entire population. At the end of the first period, policyholders will have different health states: With probability p_{aa}^1 , they will remain active; with probability p_{as}^1 , they will become sick; with probability $1 - p_{aa}^1 - p_{as}^1$, they will die. Furthermore, they will receive state dependent end-of-period income: hy , uy and 0, respectively. In the second period, they will receive no further income and have different probabilities of death for different health states, namely $1 - p_{aa}^2$ for active and $1 - p_{ss}^2$ for sick. If they remain alive, they will remain in the same health states, since, for both health states, people will certainly die at the end of the third period. The unit time interest rate is denoted by r .

The life insurer will now choose the price of the insurance policy at time zero, i.e. a face value F per unit payment of premium. The surrender value of a policy is determined by its actuarially fair present value calculated using the active health state with a discount factor δ , which can be interpreted as the difference of interest rate from lending and borrowing. Similar to Section 3, under the competitive market assumption, the life settlement company will set the settlement value simply as the actuarially fair present value of the policy, but the policyholder will need to pay an additional fixed cost K in case she chooses to settle. The policyholder, on the other hand, purchases shares of policies at the beginning of the first period, and chooses period consumptions at the beginning of each period when still alive. However, we assume that she can only settle at the beginning of the second period, whereas she chooses to partially surrender at beginnings of both the second and third period.

4.2 The Policyholder's Perspective

Based on the model and parameter assumptions above, we can derive recursive equations of the policyholder's value function. Specifically, at the beginning of the first period, we have:

$$V_1(y, a) = \max_{c,p} U(c) + \beta^{n_1} \times [p_{aa}^1 V_2(W_1 + hy, a, p) + p_{as}^1 V_2(W_1 + uy, s, p) + (1 - p_{aa}^1 - p_{as}^1) B(pF + W_1)], \quad (9)$$

such that

$$c \geq 0, p \geq 0, W_1 = (1 + r)^{n_1} \times (y - c - n_1 \times p) \geq 0,$$

where p denotes the amount of annual premium chosen by the policyholder.

At the beginning of the second period, the policyholder has two basic options: settle her policy or partially surrender it; we can treat retaining the policy and totally surrendering as two extreme cases of a partial surrender. Therefore, for the policyholder in state a , the value function can be written as:

$$V_2(W_1 + hy, a, p) = \max\{V_2^1(W_1 + hy, a, p), V_2^2(W_1 + hy, a, p)\}, \quad (10)$$

in which V_2^1 is the value function of partially surrender while V_2^2 is the value function in case of a settlement. In particular,

$$V_2^1(W_1 + hy, a, p) = \max_{c, \alpha} U(c) + \beta^{n_2} \times [p_{aa}^2 V_3(W_2, p, \alpha) + (1 - p_{aa}^2)B(W_2 + (1 - \alpha)pF)], \quad (11)$$

such that

$$c \geq 0, 0 \leq \alpha \leq 1, W_2 = (1 + r)^{n_2} \times (W_1 + hy - c - n_2 p + \alpha \delta^{n_2} \rho_a p F)$$

where ρ_a is the actuarially fair rate of surrender based on health state a , which can be calculated by mortality rates; we will give the explicit form of ρ_a in the following subsection.

Similarly, we have:

$$V_2^2(W_1 + hy, a, p) = \max_c U(c) + \beta^{n_2} \times [p_{aa}^2 V_3(W_2, 0, 1) + (1 - p_{aa}^2)B(W_2)], \quad (12)$$

such that

$$c \geq 0, W_2 = (1 + r)^{n_2} \times (W_1 + hy - c - n_2 p + \rho_a p F - K).$$

Policyholders in state s will have similar value functions with corresponding modifications on utility functions and other state-dependent parameters.

At the beginning of the third period, we can write the value function as:

$$V_3(W_2, p, \alpha) = \max_{c, \alpha'} U(c) + \beta^{n_3} \times B(W_3 + (1 - \alpha - \alpha')pF), \quad (13)$$

such that

$$0 \leq \alpha' \leq 1 - \alpha, W_3 = (1 + r)^{n_3} (W_2 - n_3 p - c + \alpha' \delta^{n_3} \rho p F).$$

Since both health types cannot survive over the end of the third period, ρ is the same for both types. Also, there is no difference in V_3 for active or sick policyholders.

Due to the existence of fixed cost K , without looking at detailed example, it's natural to induce that policyholders with higher initial wealth y are more willing to settle their policies than relatively poor policyholders. Surrendering may even be preferred to settling for those poor policyholders who are sick if K outweighs the reduced surrender value for sick policyholders. Therefore, even if the emergence of life settlement can be regarded as a tool to reduce the monopsony power of life insurer, it may only benefit policyholders who are relative rich but add more cost on poor policyholders. We will discuss this possible phenomenon in the next section.

4.3 The Life Settlement Company and the Life Insurer's Perspective

Both the life settlement company and the life insurer will obtain zero net expected profit under the assumption of complete competition in both primary and secondary life insurance markets. Therefore and due to the assumption that the policyholder can only settle at the beginning of the second period, the life settlement company will choose actuarially fair rates ρ_a and ρ_s with:

$$\rho_a = \frac{(1 - p_{aa}^2)}{(1 + r)^{n_2}} + p_{aa}^2 \left(\frac{-n_3}{(1 + r)^{n_2} F} + \frac{1}{(1 + r)^{n_2} (1 + r)^{n_3}} \right), \quad (14)$$

$$\rho_s = \frac{(1 - p_{ss}^2)}{(1 + r)^{n_2}} + p_{ss}^2 \left(\frac{-n_3}{(1 + r)^{n_2} F} + \frac{1}{(1 + r)^{n_2} (1 + r)^{n_3}} \right), \quad (15)$$

as the proper rates to pay the policyholder after paying for the second period premium $n_2 p$. In particular, notice that the equilibrium settlement value is independent of the policyholder's income and choice, but only depends on the mortality rates and life insurer's choice of face value per unit premium F .

For the life insurer, on the other hand, the calculation of its expected profit is more complicated. The insurer needs to take into account policyholder's actions and the general distribution of income $f(y)$ of policyholders. Notice that the policyholder's decision set is based on given F , so we can denote all policyholder's choices as functions of her income y and F .

For a policyholder with given y and F , from the discussion of policyholder's perspective above, we will know her optimal set of choice. Denote $p(y, F)$ as the shares of policies she will purchase, $1_a(y, F)$, $1_s(y, F)$ as the indicator functions of whether to settle or not, in which $1_a(y, F) = 1$ means the policyholder will settle her policy at the beginning of period 2 when she is in the state a and $1_a(y, F) = 0$ means she will not. Given $1_s(y, F) = 0$, let $\alpha_a(y, F)$, $\alpha_s(y, F)$, $\alpha'_a(y, F)$ and $\alpha'_s(y, F)$ be the associated partial surrender rates. Therefore, for this policyholder, the expected

profit of the insurance policy can be written as:

$$\begin{aligned}
\pi(y, F) = & n_1 p(y, F) - (1 - p_{aa}^1 - p_{as}^1) \frac{p(y, F) F}{(1+r)^{n_1}} \\
& + p_{aa}^1 \times \{1_a(y, F) \left[\frac{n_2 p(y, F)}{(1+r)^{n_1}} - (1 - p_{aa}^2) \frac{p(y, F) F}{(1+r)^{n_1} (1+r)^{n_2}} \right. \right. \\
& \left. \left. + p_{aa}^2 \left(\frac{n_3 p(y, F)}{(1+r)^{n_1} (1+r)^{n_2}} - \frac{p(y, F) F}{(1+r)^{n_1} (1+r)^{n_2} (1+r)^{n_3}} \right) \right] \right. \\
& + (1 - 1_a(y, F)) \left[\frac{n_2 p(y, F)}{(1+r)^{n_1}} - \alpha_a \frac{\delta^{n_2} p(y, F) F}{(1+r)^{n_1}} - (1 - p_{aa}^2) (1 - \alpha_a) \frac{p(y, F) F}{(1+r)^{n_1} (1+r)^{n_2}} \right. \\
& \left. + p_{aa}^2 \left(\frac{n_3 p(y, F)}{(1+r)^{n_1} (1+r)^{n_2}} - \alpha'_a \frac{\delta^{n_3} p(y, F) F}{(1+r)^{n_1} (1+r)^{n_2}} \right. \right. \\
& \left. \left. - (1 - \alpha_a - \alpha'_a) \frac{p(y, F) F}{(1+r)^{n_1} (1+r)^{n_2} (1+r)^{n_3}} \right] \right\} \\
& + p_{as}^1 \times \{1_s(y, F) \left[\frac{n_2 p(y, F)}{(1+r)^{n_1}} - (1 - p_{ss}^2) \frac{p(y, F) F}{(1+r)^{n_1} (1+r)^{n_2}} \right. \right. \\
& \left. \left. + p_{ss}^2 \left(\frac{n_3 p(y, F)}{(1+r)^{n_1} (1+r)^{n_2}} - \frac{p(y, F) F}{(1+r)^{n_1} (1+r)^{n_2} (1+r)^{n_3}} \right) \right] \right. \\
& + (1 - 1_s(y, F)) \left[\frac{n_2 p(y, F)}{(1+r)^{n_1}} - \alpha_s \frac{\delta^{n_2} p(y, F) F}{(1+r)^{n_1}} - (1 - p_{ss}^2) (1 - \alpha_s) \frac{p(y, F) F}{(1+r)^{n_1} (1+r)^{n_2}} \right. \\
& \left. + p_{ss}^2 \left(\frac{n_3 p(y, F)}{(1+r)^{n_1} (1+r)^{n_2}} - \alpha'_s \frac{\delta^{n_3} p(y, F) F}{(1+r)^{n_1} (1+r)^{n_2}} \right. \right. \\
& \left. \left. - (1 - \alpha_s - \alpha'_s) \frac{p(y, F) F}{(1+r)^{n_1} (1+r)^{n_2} (1+r)^{n_3}} \right] \right\}. \tag{16}
\end{aligned}$$

In turn, the total profit of the life insurer can thus be written as:

$$\Pi(F) = \int \pi(y, F) f(y) dy \tag{17}$$

and the life insurer chooses F^* such that $\Pi(F) = 0$. Notice that $\Pi(F^*) = \int \pi(y, F^*) f(y) dy = 0$, therefore, it must be the case that for some y , $\pi(y, F^*) > 0$ and for others, $\pi(y, F^*) < 0$. Thus, there exists a subsidy among policyholders with different wealths.

Allowing a policyholder to settle her life insurance policy is like adding a new option in the insurance contract. As the option of settling is usually preferred to the option of surrendering by policyholders, under the assumption of a competitive insurance market, the premium of insurance will necessarily increase due to the erosion of surrender profits of the insurance company. However, the usefulness of this newly added option may be significantly different for policyholders with different characteristics, such as wealth or health states. As discussed in Section 3.2, while keeping other conditions the same, policyholders with high-face-value policies or impaired health

states are more willing to settle their life insurance policies in an imperfect secondary insurance market, which means that they are implicitly taking advantage of policyholders who are poorer or healthier. Therefore, the social welfare is redistributed among policyholders. As the marginal utility is usually higher in poor and unhealthy states, the welfare effect of the secondary insurance market is theoretically indeterminable without further detailed investigation.

5 Conclusion

This paper analyzes the impact of a secondary life insurance market from different perspectives. For existing policies, we propose a two-stage model as a dynamic game between a life settlement company and a policyholder. Aside from assuming symmetric information, we investigate the impact of an information asymmetry regarding the policyholder's health state. We find that asymmetric information yields a possible explanation for the seemingly high expected rates of return advertised in the policy illustrations of a typical life settlement. The possibilities for adverse selection are particularly pronounced for relatively healthy individuals. Hence, to confine selection effects, a minimal amount of medical documentation is required leading to a considerable fixed cost within the transaction.

To investigate the repercussions of a secondary life insurance market on social welfare, we future propose a three-stage model additionally considering a life insurer. We find that in a perfect secondary market, i.e. if there are no information asymmetries or transaction costs, the existence of life settlements may be welfare-enhancing despite an increased cost of insurance and an associated front-loading: essentially, resources are shifted from healthy states early in life to relatively unhealthy states later in life where marginal utility may well be higher. This is in contrast to the conclusion by Daily et al. (2008), who believe that front-loading will lead to negative welfare effects. However, transaction costs resulting from e.g. required medical documentation may alleviate or even invert the potentially positive effects since it precludes relatively poor unhealthy policyholders with a high marginal utility from participating in life settlement transactions. The negative welfare effects for them associated with the front-loading of the policies may be close to or even dominate positive effects for wealthier individuals, so depending on welfare and health distribution, this cross subsidy from poor to wealthy policyholders may be undesirable.

While the quantitative results seem relatively robust thus far, parts of the paper require further work to confirm our conclusions. Specifically, Section 4 is incomplete in that it so far lacks numerical analysis and a more detailed discussion of the welfare effects associated with a secondary life insurance market. Moreover, a thorough discussion of the simplifying assumptions of our models, particularly in view of the policyholder's insurance choices, seems necessary to assess potential limitations of our approach.

In future research, aside from these immediate aspects, we intend to confirm our theoretical conclusions by an empirical investigation if suitable data is available.

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Appendix

A Additional Equations for Section 3.1

In this appendix, we give all remaining value functions and boundary conditions besides $V_{x,x_0}(W, a, o, n)$ which is already given in Section 3.1.

If a policyholder has already settled or surrendered her policy, she will neither be able to receive any death benefit, nor does she need to pay further premiums. Hence, she can only choose consumption for each period. Therefore, the value functions after settling or surrendering are the same, i.e., $V_{x,x_0}(W, a, 1, n) = V_{x,x_0}(W, a, 2, n)$ and

$$\begin{aligned}
V_{x,x_0}(W, a, 1, n) = & \max_c \{U(c) + \beta \times [\gamma p_{aa}^x V_{x+1,x_0}(W', a, 1, n) + \gamma p_{as}^x V_{x+1,x_0}(W', s, 1, n) \\
& + \gamma(1 - p_{aa}^x - p_{as}^x) V_{x+1,x_0}(W', d, 1, n) + (1 - \gamma) p_{aa}^x V_{x+1,x_0}(W', a, 1, u) \\
& + (1 - \gamma) p_{as}^x V_{x+1,x_0}(W', s, 1, u) \\
& + (1 - \gamma)(1 - p_{aa}^x - p_{as}^x) V_{x+1,x_0}(W', d, 1, u)]\} \tag{18}
\end{aligned}$$

$$\begin{aligned}
\text{s.t. } \quad W' &= (W - c) \times (1 + r), \\
0 &\leq c \leq W.
\end{aligned}$$

In case a shock to the bequest function has happened, the insurance policy is no longer needed by the policyholder and her bequest function $B(\cdot)$ becomes 0. Moreover, we assume this state is an absorbing state. Hence, there is no incentive for the policyholder to keep the policy, and thus she will simply compare the settlement value and the surrender value to pick the greater one:

$$\begin{aligned}
V_{x,x_0}(W, a, 0, u) = & \max_c \{U(c) + \beta \times [p_{aa}^x V_{x+1,x_0}(W', a, 1, u) \\
& + p_{as}^x V_{x+1,x_0}(W', s, 1, u) + (1 - p_{aa}^x - p_{as}^x) V_{x+1,x_0}(W', d, 1, u)]\} \tag{19}
\end{aligned}$$

$$\begin{aligned}
\text{s.t. } \quad W' &= [\max\{\rho(a, x, x_0) \times F - K, \zeta(x, x_0) \times F\} + W - c] \times (1 + r), \\
0 &\leq c \leq \max\{\rho(a, x, x_0) \times F - K, \zeta(x, x_0) \times F\} + W.
\end{aligned}$$

Similarly, we have $V_{x,x_0}(W, a, 1, u) = V_{x,x_0}(W, a, 2, u)$ and

$$\begin{aligned}
V_{x,x_0}(W, a, 1, u) = & \max_c \{U(c) + \beta \times [p_{aa}^x V_{x+1,x_0}(W', a, 1, u) \\
& + p_{as}^x V_{x+1,x_0}(W', s, 1, u) + (1 - p_{aa}^x - p_{as}^x) V_{x+1,x_0}(W', d, 1, u)]\} \tag{20}
\end{aligned}$$

$$\begin{aligned}
\text{s.t. } \quad W' &= (W - c) \times (1 + r), \\
0 &\leq c \leq W.
\end{aligned}$$

Above are all value functions for state a . For state s , we could construct value functions of exactly the same form, the only modifications being state-dependent features, i.e., we replace $p_{aa}^x \rightarrow p_{sa}^x$, $p_{as}^x \rightarrow p_{ss}^x$, $\rho(a, x, x_0) \rightarrow \rho(s, x, x_0)$ and $U(\cdot) \rightarrow \tilde{U}(\cdot)$.

State d , on the other hand, is also an absorbing state and there are no further utility in future periods. Thus, we have:

$$V_{x,x_0}(W, d, 0, n) = B(W + F); \tag{21}$$

$$V_{x,x_0}(W, d, 1, n) = V_{x,x_0}(W, d, 2, n) = B(W); \quad (22)$$

$$V_{x,x_0}(W, d, 0, u) = V_{x,x_0}(W, d, 1, u) = V_{x,x_0}(W, d, 2, u) = 0. \quad (23)$$

Let ω be the limiting age, i.e., the highest attainable age. Then, there are boundary conditions that need to be satisfied at this terminal age. For state d , there are no iterations in the value function, so we just need boundary conditions for state a and s . Notice that at age ω , there is no difference about the survival probability between an active policyholder or a sick policyholder. All boundary conditions are shown as follows:

$$V_{\omega,x_0}^i(W, a, 0, n) = \max_c \{U(c) + \beta V_{\omega,x_0}(W', d, i, n)\}, \quad i = 0, 1, 2. \quad (24)$$

With different constraints:

$i = 0$:

$$\begin{aligned} \text{s.t.} \quad W' &= (W - c - \alpha(x_0) \times F) \times (1 + r), \\ 0 &\leq c \leq W - \alpha(x_0) \times F. \end{aligned}$$

$i = 1$:

$$\begin{aligned} \text{s.t.} \quad W' &= (W - c - K + \rho(a, \omega, x_0) \times F) \times (1 + r), \\ 0 &\leq c \leq W - K + \rho(a, \omega, x_0) \times F. \end{aligned}$$

$i = 2$:

$$\begin{aligned} \text{s.t.} \quad W' &= (W - c + \zeta(\omega, x_0) \times F) \times (1 + r), \\ 0 &\leq c \leq W + \zeta(\omega, x_0) \times F. \end{aligned}$$

$$V_{\omega,x_0}(W, a, 1, n) = V_{\omega,x_0}(W, a, 2, n) = \max_c \{U(c) + \beta B(W')\} \quad (25)$$

$$\begin{aligned} \text{s.t.} \quad W' &= (W - c) \times (1 + r), \\ 0 &\leq c \leq W. \end{aligned}$$

Similar, for state s , the only modifications are state-dependent features, i.e., replace $\rho(a, \omega, x_0) \rightarrow \rho(s, \omega, x_0)$ and $U(\cdot) \rightarrow \tilde{U}(\cdot)$.

If the shock to the bequest function has happened at the beginning of the last period, then the policyholder will consume all her wealth. Therefore, we have:

$$V_{\omega,x_0}(W, a, 0, u) = U(\max\{\rho(a, \omega, x_0)F - K, \zeta(\omega, x_0)F\} + W), \quad (26)$$

$$V_{\omega,x_0}(W, s, 0, u) = \tilde{U}(\max\{\rho(s, \omega, x_0)F - K, \zeta(\omega, x_0)F\} + W), \quad (27)$$

$$V_{\omega,x_0}(W, a, 1(2), u) = U(W), \quad (28)$$

$$V_{\omega,x_0}(W, s, 1(2), u) = \tilde{U}(W). \quad (29)$$