

A bottom-up approach to Pay-As-You-Drive car insurance*

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Abstract

Pay-As-You-Drive (PAYD) is a concept in private automobile insurance under which premiums are quoted per mile. It is currently being tested by insurers world wide. With its increase in marginal driving costs Pay-As-You-Drive may present a politically feasible way to reduce driving (around 9%, according to recent studies) all the while saving most people money and increase fairness. The literature so far has analyzed PAYD with a representative agent approach, but concedes that both drivers' heterogeneity and changes to caretaking incentives could significantly affect their predictions. To examine these potential side-effects I propose a bottom-up approach to compare individual driving behavior under various types of insurance pricing. One of the key findings is that when switching from standard to per-mile premiums drivers will for the most part reduce their driving but also exert less effort. Moreover there are considerable heterogeneity effects among drivers. In particular I find that infrequent drivers may actually increase their driving, which provides current non-drivers with more incentives to start driving than previously thought. In addition, I show that individual incentives to reduce driving decline when everybody else drives less. For these reasons aggregate mileage reduction and social benefits of PAYD may actually be less than the current estimates. Lastly I find that monitoring driving behavior will provide drivers with economic incentives to drive more carefully only if, in the absence of monitoring, they would not see their premiums increase too much after being involved in an accident.

Keywords: Pay-As-You-Drive, Per-mile pricing, Automobile insurance, Moral Hazard, Driving externalities, Event Data Recorders.

*This is currently a working paper. Comments and suggestions are very much appreciated.

1 Introduction

Ever since the first policy offering liability coverage was issued by an English company in 1895, premiums for automobile insurance have been quoted per period and have been largely independent of how much one drives. However, with being an additional mile on the road, not only do I pose more costs to my insurer (by increasing the chance of causing an accident¹), but I also impose costs on other drivers (respectively their insurers) – who would not have hit my car had I not been on the road in the first place – and society (through increased congestion, road maintenance, pollution, carbon emissions, oil dependence, etc.).

The latter two, the costs to other drivers and society that is, are externalities of driving that a profit-maximizing insurer does not care about². I will address these costs in section 1.2. The focus of this paper lies however on the inefficiencies between a driver and her insurer which I will explain in more detail first. While Pay-As-You-Drive is ...

Nevertheless, studies show that the *social* benefits of Pay-As-You-Drive will yield considerable externality benefits

1.1 The Moral Hazard Effect in Auto Insurance

Parry Walls and Harrington (2007) estimate that in 2000 U.S. accident costs aggregate to \$433 billion. The average American spends \$817 per year and vehicle on car insurance, according to National Association of Insurance Commissioners data³ [IFB (2009)]. A driver who wants liability, comprehensive *and* collision coverage would need to pay a U.S. average of \$936 [IIM (2008)]. According to IFB (2009) the most expensive U.S. city is Detroit, MI where drivers pay an average of \$5,072 to insure their car⁴.

Litman (2009) shows that low-income vehicle-owning households devote about five times the portion of their household budget to vehicle insurance as compared to high-income households. Litman claims that for many low-income households insurance is by far the largest cost of driving or the main reason they cannot afford a vehicle. Moreover, the high insurance premiums are also a major reason why an estimated 14.6% of U.S. motorists are driving without insurance [IRC (2006)].

The presence of insurance introduces inefficiencies: Pauly (1968) finds an economic explanation for the presence of (*ex-ante*⁵) moral hazard in insurance markets. Cohen and Dehejia (2004) find empirical evidence of moral hazard costs in the auto insurance market by investigating incentive effects of auto insurance,

¹or simply having an accident, if in a no-fault insurance system

²Actually, the insurer does care about other drivers' subsequent reduction of accident risk *if* they are insured with himself. That is he cares about whether or not a driver takes the bus only to the extent that he is covering the local auto insurance market. This should technically increase the incentives for large insurers to induce less driving because their benefits would be greater than 100%. Let us for simplicity of argument assume we are considering a small insurer here.

³Around 77% of insured drivers purchase comprehensive coverage in addition to liability insurance, and 72% buy collision coverage.

⁴Assumes \$100,000 / \$300,000 / \$50,000 liability limits, collision and comprehensive with \$500 deductibles, and \$100,000 / \$300,000 uninsured coverage.

⁵Throughout this paper, *moral hazard* refers to insurance weakening the incentives for an insured to engage in accident prevention activities, as opposed to "ex post moral hazard" which refers to incentives for an insured to undertake actions that alter the claim amount *after* an accident has occurred. There is evidence of ex-post moral hazard in the auto insurance market as well [Cummins and Tennyson (1996)].

compulsory insurance laws and no-fault liability laws on driving behavior and traffic fatalities⁶. These studies show that despite deductibles and coinsurance, despite the risk of personal injury to themselves and their passengers, insurance still provides drivers with considerable incentives to drive more frequent and less careful than is efficient.

So far, insurers try to determine accident risk and thus insurance premiums by categorizing drivers based on basic characteristics such as age, sex, education, zip code, etc. Bond and Crocker (1991) show how an insurer can defer more information regarding a driver's risk type from looking at voluntary consumption of the driver; for instance driving a sports car might indicate that the driver is rather risk loving and more likely to speed. Another indicator may be driving record. However, accidents happen so infrequently that a driving record does not allow for much conclusion, especially for younger drivers [Bordoff and Noel (2008)]. All in all, experts agree that the present auto insurance system still includes large cross-subsidization from low-risk to high-risk drivers, as insurers have little information about the drivers' risk types. The lack of adverse selection in the insurance market, as noticed for example by Dionne Gouriéroux and Vanasse (2001), can be attributed to drivers not knowing their own risk type any better than the insurer, rather than insurers having near-perfect information about drivers' risks.

A potential (partial) solution to these problems of insurance inaffordability, uninsured driving, unfairness and inefficiency dates back to 1968 and economist William Vickrey who proposed *usage-based* car insurance to deal with the inefficiencies and inequalities of the auto insurance market. Vickrey (1968) explains the accident externality with the difference between marginal and average cost of driving and suggests to sell insurance with tires or gasoline. The latter has become known as *Pay-At-The-Pump* auto insurance and is more closely described by Tyler and Hoffer (1973). Sugarman (1993) proposes the *California Vehicle Insurance Plan* as an alternative to the present insurance system, thereby relying on *Pay-At-The-Pump* as the primary way of collecting premiums.

Now, 40 years later, Vickrey's vision is about to become reality⁷, and does so apparently in the form of *Pay-As-You-Drive* (PAYD), also known as *Pay-By-The-Mile*, auto insurance⁸. Under this concept insurers quote premiums per mile instead of per period, verify some way how many miles the car has been driven and then charge the driver ex post the product of her per-mile premium and miles driven.

Driving data from the state of Texas indicates that the number of crashes per vehicle tends to increase with annual mileage⁹, providing empirical justification for PAYD policies [Progressive (2005)].

Edlin (2003) computes potential benefits of charging uniform per-gallon premiums, per-mile premiums (in both cases so that the insurer breaks even) and *optimal* per-mile premiums (where premiums are taxed to allow for accident externalities) via several models. Edlin provides state-by-state estimates of premiums and driving reduction, and finds a U.S. average premium of 4 cents per mile and a national reduction in

⁶In particular the authors find that no-fault liability laws lead to an increase in traffic fatalities of about 6%.

⁷Several insurers around the globe are currently testing such usage-based pricing products. For more information on current implementations, see section 4.1.

⁸In the literature, the term "Pay-As-you-Drive" is typically used as an umbrella term for all kinds of usage-based auto insurance. For the purpose of this paper however, PAYD shall stand synonymous for "per-mile pricing" and "Pay-By-The-Mile", since it is this subcategory of usage-based insurance that is most likely to enter the market in the near future.

⁹This increase is however sub-proportional — this can be attributed to frequent drivers being more experienced behind the wheel (and maybe self-aware bad drivers drive less because they do not want to put their lives at risk by getting on the road), so that they have less per-mile but still a higher aggregate risk of having an accident.

vehicle miles traveled (VMT) of approximately 10% under (regular) per-mile premiums, with tremendous heterogeneity among states¹⁰. Due to heterogeneity in fuel efficiency uniform per-gallon premiums reduce VMT by only around 8%. Optimal per-mile premiums would trigger a VMT reduction of circa 15%. The author estimates national accident costs savings net of lost driving benefits of \$12.7 billion (\$75 per vehicle), \$10.4 billion (\$61) and \$15.3 billion (\$91) per year for each of the three premium categories. In addition, switching to per-mile premiums would yield substantial benefits from reduced fatalities (ca. 6600 per year) and reduced congestion (estimated to be worth between \$4.2 billion to \$9.4 billion annually), not to mention gains from reduced pollution, oil dependence, etc.

Parry (2005) compares the welfare gains of PAYD auto insurance with the benefits from a comparable gasoline tax. He applies a utility maximization approach having a representative agent in a competitive auto insurance environment choose the number of vehicles, fuel efficiency, mileage and composite good consumption. The author projects a 6.5 cents per mile charge for insurance if PAYD were fully implemented, leading to a reduction of gasoline demand of 11.4 billion gallons, that is 9.1%.

Bordoff and Noel (2008) extend the analysis of Parry (2005) by using data on vehicles, driving and household characteristics from the 2001 National Household Transportation Survey. Bordoff and Noel estimate average per-mile premiums of 6.6 cents and a national reduction in driving of 8%. They attribute their lower estimates for reduction in driving with the substantial increase in gas prices between 2005 and 2008, so that the increase in marginal driving costs through per-mile insurance premiums has now less of an effect on driving behavior.

Two real-world experiments confirmed these estimates of an 8 to 10% VMT reduction, although these results have to be viewed with caution due to small sample sizes [Cambridge Systematics (2006), Progressive (2007)].

Shavell (1980) shows that under a strict liability system, where drivers must pay whenever they are involved in an accident, a driver will choose an efficient level of both mileage and caretaking. Under a negligence system however, where drivers only pay for an accident that they caused, a driver will choose an efficient level of caretaking but she will drive more than would be efficient as she is not internalizing the costs of driving another mile, only the costs of driving recklessly¹¹. To generalize the idea: a driver will behave efficiently (regarding both mileage and caretaking) if and only if she is internalizing the full costs of her driving behavior.

In order to make drivers select *efficient* levels of mileage and caretaking, insurance premiums need to reflect the driver's actual accident risk in regards to how her driving affects her insurer. In particular this requires the insurer to obtain full information on the driver's risk in the first place. This can only be achieved by monitoring her driving at any time and charging her appropriately ex post for the accident risk that she presented with her driving (but independent of whether or not she actually had an accident during the past period). New technologies – commonly known as *Event Data Recorders* (EDR) – have made that option available, although with certain costs and drawbacks. I will explore this feature in more detail in section 1.3. We can note however that under monitoring the presence of insurance is no longer causing moral hazard

¹⁰New Jersey: 6.5 cents per mile, yielding an 18% reduction in VMT. Wyoming: 1.8 cents and 4.5%.

¹¹Note that Shavell's arguments of efficiency are based on the absence of insurance which – as mentioned earlier – would induce inefficiencies on its own.

issues à la Pauly (1968) while still relieving drivers from their risk-aversion as before. All risk a driver imposes on her insurer is directly passed back to her. She will thus choose mileage and caretaking exactly at the first-best level.

In comparison, per-mile pricing without monitoring would make a driver internalize the costs she imposes on her insurer by driving frequently but not the costs from driving carelessly. She will therefore choose a first-best level of mileage but exert less effort than under first-best¹².

However, monitoring may never be everybody's darling, given its privacy concerns. Moreover, the technology has only been available for a few years and is still rather expensive.

In 2001, NHTSA (2001) notes: "Studies of EDRs in Europe and the U.S. have shown that driver and employee awareness of an onboard EDR reduces the number of crashes by 20 to 30 percent, lowers the severity of such crashes, and decreases the associated costs." The National Highway Traffic Safety Administration projects that EDRs will not only make the auto insurance market more efficient, but also improve safety significantly and assist in reconstructing crashes.

In my analysis (sections 2 and 3) I will therefore compare driving behavior (in terms of mileage *and* effort) for three types of insurance pricing: The standard case of per-period premiums, per-mile premiums with mileage verification via odometer readings, and premiums that fully reflect the driver's true risk type, as obtained under monitoring using an EDR.

1.2 Driving Externalities

Edlin and Karaca-Mandic (2006) analyze externalities from car accidents using panel data on state-average insurance premiums and loss costs. They estimate that in U.S. states with low traffic density these externalities are almost insignificant, while every additional average driver in California would increase total statewide insurance costs of other drivers by \$1725 - \$3239 annually. The authors predict that a Pigouvian tax correcting for these accident externalities could raise \$66 billion per year in California, and \$220 billion per year nationwide. Bordoff and Noel (2008) estimate external auto insurance cost savings from a switch to per-mile premiums of \$21 billion annually, that is \$93 per vehicle.

Edlin (2003) notes that insurers are not internalizing all benefits that PAYD has to offer. However, the insurer also benefits when drivers sign up for PAYD with a competitor. While larger insurers have more incentives to start introducing PAYD due to externality benefits, when PAYD is commonly available and all drivers are free to choose their insurer¹³, then the insurer will simply receive as many externality benefits from other drivers as he loses through his own drivers due to not owning a bigger share of the market. Thus in principle when drivers reduces mileage due to per-mile premiums, the benefits from reduced accident risk for others are internalized by insurers, but the benefits to society can be fully appreciated by society only. This is also why in my model I assume for simplicity of expression that the market is covered by a single competitive

¹²As I show in section 2 the insurer can increase drivers' efforts (potentially above first-best level) by making premiums depend more strongly on recent driving record, but this would come at the expense of the driver as it exploits her risk aversion rather than more accurate information about her type, and partly negate why there is car insurance in the first place.

¹³Since premiums are quoted before the driver decides with whom to insure, the Bertrand-competitive outcome will be perfect competition.

insurer¹⁴.

Therefore, PAYD will make insurers realize some externality gains due to reduced accidents by other drivers. However, in order to actually account for the *full* externality costs on other drivers, Finsinger and Pauly (1990) analyze a system of "double liability" where in a two-car accident both drivers pay the full costs of the accident, that is the damage to both their own and the other driver's car¹⁵. The double liability rule would make each driver internalize the full accident loss and therefore choose the first-best level of driving (between the driver herself and all other drivers on the road). Again, in the presence of insurance (now with roughly twice the premiums as usual) the moral hazard effect will bring mileage and caretaking away from their first-best levels.

Finally, making drivers internalize the cost of the *social* harm caused by their activities "would require a set of optimized user fees specifically calibrated to capture each externality", like for example road tolls, gasoline taxes and – potentially – taxes on per-mile insurance premiums [Bordoff and Noel (2008)].

In 2007, the average peak-period traveller spent 36 extra hours on the road due to urban traffic congestion. This wasted time is valued by Schrank and Lomax (2009) at \$87 billion, which exceeds for instance the size of the entire annual U.S. federal transportation budget. Bordoff and Noel (2008) estimate that a driving reduction of 8% would save Americans \$13.3 billion, that is \$58 per vehicle. They also estimate annual savings of \$3.3 billion from reduced local pollution, \$2.5 billion from reduced carbon emissions, and \$5.6 billion from reduced oil dependence. Total gross benefits (net of lost driving benefits) of switching to per-mile premiums accumulate to \$59 billion per year nationwide, that is \$257 per vehicle. These are substantially larger than the \$7.7 billion (\$34 per vehicle) of individual insurance cost savings net of lost driving benefits. While monitoring costs may exceed these \$34, they certainly fall way short of the total social benefits, which provides justification for government subsidies. Bordoff and Noel (2008), for instance, propose not only an increase in government funding of PAYD pilot programs, but also a \$100 tax credit from the federal government for each new PAYD policy.

Parry (2005) finds that an increase in the gasoline tax by 27 cents per gallon (from currently 18 to 45 cents) would yield an equivalent fuel reduction as a nationwide switch to per-mile premiums, but would achieve only 32% of the welfare gains of PAYD (\$19.3 billion per year). This is because mileage-related externalities (congestion, accidents, local pollution – 12 cents per mile) are roughly ten times as large as fuel-related externalities (carbon emissions, oil dependency – 24 cents per gallon, that is 1.2 cents per mile for a vehicle that drives 20 miles per gallon).

Neither of these suggestions to reduce driving can be implemented smoothly it seems. To capture the externalities to other drivers and society the government would have to intervene and increase driving costs artificially, for instance via a gas tax. On the other hand, per-mile pricing – which addresses the mileage externality effect between a driver and her insurer – is a fundamentally new insurance product that requires a sophisticated pricing structure which needs to meet the strict requirements of actuaries and insurance

¹⁴ Assume there is more than one insurer offering PAYD. If at per-mile premium p_1 an insurer expects to make strictly positive profit he will offer premium $p_2 < p_1$ to attract more customers. His competitors need to follow, until eventually all insurers offer a per-mile premium that makes them break even. This yields the same equilibrium premium, driver-reaction and benefit-structure as assuming a single insurer who is breaking even as well.

¹⁵ This system has however a serious drawback in its implementation as drivers have incentives to not report the accident and instead renegotiate with each other and pay the full cost of the accident just once combined instead of both individually.

regulators. However, with the current auto insurance system of per-period premiums in place, a large amount of infrequent drivers subsidize a small amount of frequent drivers and most households (ca. 63%, [Bordoff and Noel (2008)]) will save money under PAYD. Moreover per-mile pricing increases fairness by removing these subsidies. Therefore, Pay-As-You-Drive has an advantage on the legislative end, and simply requires the innovative spirit of insurers¹⁶. In fact, the government even has reasons to subsidize per-mile pricing: while nothing about PAYD directly addresses the other two externality effects, by inducing people to drive less, PAYD also provides considerable indirect benefits to "other drivers" (by reducing their chances to have an accident) and "society" (by reducing congestion, oil consumption, road usage, carbon emissions, etc.). PAYD may thus present a politically feasible way to reduce driving while actually saving the average driver money and increase fairness.

1.3 How can insurers verify mileage?

In order to implement per-mile pricing, an insurer will need to find a way to verify how many miles a car was driven. Relying on drivers reporting their mileage truthfully may not be the most successful business strategy. A list of how insurers worldwide are testing and/or implementing the new product can be found in the appendix, section 4.1. In principle all approaches can be categorized into two categories: manual vs. electronic verification.

Manual verification involves odometer readings of some sort. This could be done by a certified mechanic, or by the driver herself, with random periodic checks by an insurance representative. A possible alternative currently applied is to have drivers pre-pay for their miles. The only additional costs would be to get the odometer reading done, and the time a driver is involved in that.

Electronic verification requires an event data recorder (EDR) being installed in the driver's car. Such a device can record the location of the car at any time and thereby compute mileage. In addition, some EDR models can also observe *how* the car is driven: sudden breaks and speeding, for instance, are indicators of a high-risk driver. If the insurer gains such information, a whole new dimension of car insurance becomes available. Not only does it allow insurers to near-perfectly identify a driver's accident risk and subsequently charge her fair premiums¹⁷, it also gives the insurer control of the caretaking externality, as discussed earlier. Strauss and Hollis (2008) examine the welfare effect of having a single insurer being better informed about drivers' risk types than other insurers by using an EDR, and determine that in the absence of adverse selection this information asymmetry would be welfare reducing.

Filipova and Welzel (2005) on the other hand find that an ex post (that is after an accident has occurred) revelation of information about the driver's risk type or exerted level of effort is always welfare enhancing if the reason for information asymmetry is moral hazard, and is also true under adverse selection unless when bad risks under self-selecting contracts receive informational rent.

As a drawback, many individuals view monitoring as an invasion of privacy. Details of their concerns as well

¹⁶The only type of government intervention needed to get PAYD started is to remove regulation that prohibits – explicitly or implicitly – the implementation of per-mile pricing. I provide an overview of such regulation in section 4.3.

¹⁷As Hollis and Strauss (2007) observe, "event data recorders (EDRs) and telematic technology including GPS units (...) provide a means for automobile insurance companies to discover efficient estimates of the expected losses that automobile drivers will incur."

as an innovative technology called *PriPAYD*, that is "privacy-friendly Pay-As-You-Drive" auto insurance, will be presented in section 4.4. Litman (2008) concludes that GPS-based pricing should be offered only in addition to simple odometer-based pricing, so consumers can choose the option they prefer. Offering GPS-based pricing only could significantly increase program costs and raise privacy concerns, and thus greatly reduce the potential market.

As compared to odometer readings, monitoring HOW a car is driven would further increase efficiency and welfare as bad drivers will be charged higher per-mile premiums and will thus reduce their driving more than good drivers, thereby decreasing accident rates.

If insurers monitor WHERE and WHEN a car is driven, they can discriminate by territory and time of driving. Drivers would have to pay more for driving during business traffic time in inner cities and for driving at nighttime as compared to driving at noon in the countryside. This provides drivers with incentives to avoid times and areas of high accident risk, and thus help reduce accident rates as well. In my model I do not account for heterogeneity of territories and times of driving, so that potential estimates might be too conservative.

Bordoff and Noel (2008) list several currently available devices that could be used to electronically monitor mileage (and potentially more). Once PAYD becomes available to the general public and EDRs will be mass-produced, their price should go down significantly. Annual monitoring costs should then be easily outweighed by the benefits from learning a driver's true accident risk, especially in a competitive market.

1.4 Contribution to the Literature

Pay-As-You-Drive car insurance is currently in a developing stage and still a few years away from its potential breakthrough. And while economic models are rather scarce, PAYD is discussed intensely on a more basic level from various perspectives by academics, practitioners and interested consumers alike. Hardly anyone questions the benefits of PAYD as described by Edlin (2003) and others, and there seems to be a big excitement and eagerness among many regarding the introduction of this product to the general public. However, there is considerable discussion circling the academic literature, newspaper and journal articles and the internet regarding the feasibility of implementing per-mile premiums. The main concerns are regulation and privacy issues (regarding the use of EDR devices) which I will discuss in sections 4.3 and 4.4, respectively. Section 4.2 provides an overview of other points of critique regarding PAYD and its implementation.

The current literature analyzes PAYD with the purpose of estimating the aggregate mileage reduction and subsequently the potential benefits to insurers as well as society [Edlin (2003), Parry (2005), Bordoff and Noel (2008)]. Thereby, the authors use a "macro" approach with a representative agent. However, there are several issues surrounding individual driving behavior under Pay-As-You-Drive that are not well understood yet, but that might significantly affect these estimates.

- Heterogeneity of drivers. As I show in section 2, a switch to per-mile premiums will affect good drivers differently than bad drivers and frequent drivers differently than infrequent drivers. For instance, frequent drivers have more incentives to reduce driving under PAYD, so that the reduction in mileage may be higher on aggregate than for an average driver (as considered by Edlin (2003) and

others). At the same time however, since frequent drivers also tend to be better drivers, the average driver on the road is now of higher risk than under standard pricing. On the other hand, especially when the insurer is monitoring her, a "bad" driver will be charged higher premiums and is thus given more incentives to reduce her mileage, as compared to a better driver who gets the same utility out of driving. Not to mention that under certain conditions monitoring will induce people to drive safer.

- Caretaking effort. As the NHTSA (2001) study¹⁸ shows, a driver's effort to prevent accidents can affect her accident risk significantly, even in addition to the effort she already exerts in view of the costs and potential injuries resulting from an accident. Therefore caretaking should play an important role in the analysis of PAYD. In particular I find that drivers will reduce their efforts under per-mile pricing, so that the reduction in accident costs might be lower than estimated.
- Type of mileage verification. As our analysis in section 2 indicates, individuals adjust their driving behavior based on how much information is revealed to their insurer.
- Premium adjustment in response to aggregate driving reduction. Edlin (2003), Parry (2005) and Bordoff and Noel (2008) all treat per-mile premiums as the ratio of total accident costs and aggregate miles driven, both measured under standard insurance. In particular they do not allow for premiums to adjust to the reduction in individual accident risk when fewer cars are in the streets. In a competitive auto insurance market, this would lead to a reduction in per-mile premiums which in turn gives people incentives to drive more and thus offsets some of the initial mileage reduction from increasing marginal costs of driving.
- Income effects. In section 2 I show that the most infrequent drivers will actually increase their driving under per-mile pricing. This apparently offsets the increase in marginal cost of driving under PAYD as they allocate some of the money they save on insurance costs to more driving. Similarly, very frequent drivers will *further* reduce their driving under PAYD due to their overall increase in insurance costs.
- Including new drivers. Our analysis allows us to contradict an effect proposed by Parry (2005) who suggests that PAYD may not necessarily introduce new cars and drivers to the streets (despite reduced costs of holding a car) because drivers also get less use out of having a car. As this paper shows, this is actually not true for infrequent drivers, due to the income effect. And if we assume that if people do not have a car it is because they would not drive enough to justify the total costs of holding a car, then a reduction in these costs (as is the case under PAYD) will in fact increase the number of drivers and cars. Another argument that current estimates of aggregate mileage reduction might be overstated.

While the analyses of Edlin (2003), Parry (2005) and Bordoff and Noel (2008) provide very valuable insights into PAYD and meaningful estimates of its overall benefits, we must extend their approach to enhance our understanding of this new concept. To assume that all these effects will more or less offset each other is a gross simplification that will hardly convince any actuary or regulator who is asked to sign off such a new product.

¹⁸20 to 30% reduction of accidents in the presence of an onboard EDR, see above.

At present, insurers worldwide are testing PAYD (in various forms), partly in collaboration with or subsidized by government agencies¹⁹. Now that more and more data on driving behavior under per-mile pricing will be available, more sophisticated economic models are required to assist insurers in their quest to understand the data and introduce the new product, to convince actuaries and regulators of its transparency and shareholders of its profitability. This paper is intended to provide a first step towards that goal.

Conceptually, this paper is most closely related to Edlin (2003), Parry (2005) and Bordoff and Noel (2008), although with several distinct differences: For one, I am interested in individual driving behavior and therefore use a "micro" or "bottom up" approach with heterogeneous agents. In particular, drivers differ in their valuation of driving and their accident risk. Moreover, I include the driving effects described earlier (with the exception of the "new driver effect") to allow for a more accurate analysis of PAYD.

Section 2 is designed to analyze individual driving behavior, that is mileage and accident prevention effort, under different forms of insurance pricing: the standard case of per-period premiums, per-mile premiums with mileage verification via odometer readings, and finally per-mile premiums where the insurer monitors the driver perfectly using an EDR device. Using a one-period model I find that when switching to per-mile premiums most drivers will not only drive less but also exert less effort. And drivers will have economic incentives to increase their effort under monitoring (relative to odometer-based per-mile pricing) only if they anticipate that per-mile premiums in the non-monitoring case do not depend too much on recent accident history.

Section 3 extends the model to a dynamic framework, allowing to more accurately determine a driver's premium based on personal characteristics as well as driving record. By aggregating over individual driving behavior we can thus determine numerically the total reduction in mileage under each pricing scenario, while implicitly accounting for heterogeneity of drivers and caretaking efforts. The model determines the true equilibrium per-mile premium in response to drivers changing their driving behavior under PAYD. For that matter this approach may yield more accurate estimates of mileage reduction under PAYD as compared to the present literature, and allow for a more detailed analysis of its benefits, for instance with respect to accident reductions.

At the same time this paper may provide a starting point of discussion – by academics and practitioners alike – towards more sophisticated and realistic per-mile insurance pricing models. And while I believe that we need to acquire a better understanding of PAYD before it can be introduced to the general public, I am fully convinced that the aggregate benefits of switching to per-mile premiums, as shown by Edlin (2003) and others, are likely to be tremendous.

2 Individual driving behavior under Pay-As-You-Drive car insurance

2.1 Description of the model

In order to analyze the changes in driving behavior when switching to per-mile pricing, I propose a static economic model comparing insurance premiums of the following types:

¹⁹An overview of several noteworthy programs can be found in the Appendix, section 4.1.

- **(Type S)** The insurer is offering standard insurance with per-period premiums, using basic characteristics (and driving record) as proxies for the driver's accident risk. (Sections 2.3 and 3.2)
- **(Type O)** The insurer observes mileage (e.g. using odometer readings), that is he can quote per-mile premiums, which are again determined according to basic characteristics (and driving record). (Sections 2.4 and 3.3)
- **(Type M)** The insurer is monitoring all drivers, observes mileage and driving style, that is he can quote per-mile premiums based on the driver's true accident probability (e.g. using EDR technology). (Sections 2.5 and 3.4)

The insurance market is competitive and populated by a single risk-neutral insurer ("he"). The insurer only offers full insurance under all types of contracts, and drivers do not face any disutility from having an accident or being caught speeding, other than the non-monitoring insurer being notified and increasing premiums (due to updated beliefs regarding the true accident risk)²⁰.

The economy consists of a continuum of heterogeneous, risk-averse drivers ("she") of unit mass, who are characterized by the following variables:

- θ_q , representing the basic characteristics (age, sex, zip code, experience, etc.) of the driver.
- Y , her (exogenous) income for the period.
- g , the (exogenous) price of gas that allows her to drive one mile.
- θ_m , describing her valuation of driving frequently. (Everything else equal, a larger θ_m indicates a higher utility from driving.)
- θ_a , describing her (true) per-mile accident risk. (Everything else equal, a larger θ_a indicates a more risky driver.)
- P , describing her valuation of locational privacy.

For simplicity, let us summarize a driver's risk-related variables to $x \equiv (Y, g, \theta_m, \theta_a, P)$.

Note that x provides all necessary information to compute a driver's true accident risk, whereas θ_q only contains partial information thereof. In particular, θ_q contains the very information about the driver that is available to a non-monitoring insurer (excluding her driving record, if applicable)²¹. For simplicity of analysis and argumentation I will assume that under Type O pricing the insurer will not use observed mileage to infer the driver's risk group. That is, as will be explained below, a driver's premiums are still based on a mixture of driving record and average premium for a cohort, where the cohort will consist of all drivers with

²⁰This assumption is not as strict as it may seem at first. Fines, deductibles and coinsurance provide incentives for drivers to exert effort. However, what ultimately matters for the premium is the driver's true accident risk, whether he's good and careless or a bad but careful driver doesn't matter (and is also not reflected in accident data). The purpose of the model is to illustrate the *changes* in driving behavior under a different pricing scheme, NOT to determine what part of a driver's accident risk can be attributed to talent and what part is effort.

²¹In practice the insurer might be able to further specify the distribution of x . For instance, since he knows what type of car is insured and where the driver lives, he should also be able to determine g quite well.

the same θ_q , independent of how much they drive. That being said, even though θ_q does not have a direct effect on the driver's behavior, it does affect how she is perceived by a non-monitoring insurer which in turn determines the premium she will be charged.

From the insurer's perspective, let us denote X and Q as the random variables corresponding to x and θ_q . Let Q follow the cumulative distribution function $F_Q(\theta_q)$. Conditional on $Q = \theta_q$, let X follow the cumulative distribution function $F_{X|Q}(x|\theta_q)$.

I assume that a driver knows her own risk type x ²².

A driver gains utility from both consumption and driving. There are no savings allowed, so that consumption will be income net of payments for gas and insurance. For simplicity, let utilities from driving m miles and consuming $\$z$ be additively separable with utility functions $V(m)$ and $U(z)$ respectively. The driver also endures a per-mile disutility of effort of $\Psi(e)$.

Each driver chooses mileage m and effort e optimally in order to maximize her expected utility.

A monitoring insurer has perfect information regarding the driver's accident risk and can price her accordingly. A non-monitoring insurer on the other hand needs to rely on the individual's basic characteristics and driving record to get a better estimate of the fair premium.

Every traffic incident a driver is involved in results in the (non-monitoring) insurer updating his evaluation of the driver's accident risk. This typically leads to an increase in future premiums. Conversely, if you manage to drive without an accident for a longer period of time, your premium will go down. In a static model driving record cannot be carried over to the next period. Therefore, I assume premiums are collected at the end of the period, based on the updated accident risk a driver imposed on her insurer. More specifically, premiums depend to a share of α_i ($i \in S, O$) on an average premium of q for the driver's cohort (all drivers with same basic characteristics θ_q) and to a share of $1 - \alpha_i$ on the losses she caused her insurer during the past period. In other words, in this one-period setting a driver pays a basic premium that is common to all drivers of that cohort – under Type O proportional to mileage – and in addition pays a share of all accidents costs she caused during that period²³. By way of defining the premium function, the accident payment structure is the same under per-mile and per-period pricing. But of course, the expected number of accidents is also linearly increasing in mileage, as discussed below. In particular, I assume that under Type O the insurer does not further categorize by mileage, that is he pools drivers only by basic characteristics θ_q , so that premiums are based on the same information as under Type S. To simplify the analytical comparisons in this section I will further assume $\alpha_O = \alpha_S$, and from now on denote it as α . Many large U.S. car insurance are now offering "accident forgiveness" programs, which might reflect an insurer's unwillingness to raise premiums to an actuarially fair level. It may also indicate that in practice α is to some extent discretionary rather than a best estimate obtained from a deeply sophisticated econometric analysis of past premiums and driving behavior.

²²However, low risk drivers cannot signal their type (in the sense of Rothschild and Stiglitz (1976)) since only full coverage insurance is available.

²³In practice, under a multi-period setting, the insurer would increase future premiums and thereby collect part of the accident costs back from the driver. For simplicity I assume here that the driver simply pays the present value of these future premium increases right away.

In the case of monitoring (that is type-M pricing) the premium would reflect her true accident risk given her choice of mileage and effort (independent of how many accidents she had).

As for the driving record, there are three kinds of *driving incidents* that he observes:

- A **1-car accident** caused by the driver, resulting in a fixed loss of $L/2$.
- A **2-car accident** caused by the driver, resulting in a fixed loss of L .
- A **traffic violation** (e.g. speeding, DUI) that is detected and reported by the police, resulting in a loss of 0 .

This is of course assuming a tort insurance system. In a no-fault environment, *any* 2-car accident will result in an expected loss of $L/2$, and the following analysis would be very similar.

Drivers are homogeneous to the extent that the respective probabilities conditional on a driving incident happening are the same for all drivers. In the case of standard insurance, we denote them by γ_1^S for 1-car accidents and γ_2^S for 2-car accidents, and $1 - \gamma_1^S - \gamma_2^S$ for traffic violations. Also, let λ_S denote the probability of any given driver having a driving incident when driving one mile (this probability of course is a function of the driver's type and effort: $\lambda_S(x, e)$).

For other types of insurance pricing these probabilities may be different due to the change in aggregate mileage and the externality effect of driving, as analyzed in section 2.2.

All types of driving incidents carry the same information regarding the driver's accident risk. An individual's driving record will therefore be based on the aggregate number of those incidents in period t , denoted by N_t . Since all losses are borne by the insurer, the total payment made in period t on behalf of a driver is thus $N_t L_i$, with L_i being defined below as the expected loss given that an incident has occurred under Type- i pricing, $i \in S, O, M$.

Assuming driving incidents occur independent of each other (over time as well as among drivers), N_t follows a Poisson distribution with mean $\lambda_i(x, e_t) \times m_t$, $i \in S, O, M$.

Finally, let us assume the following specifics for our functional forms:

Assumption 2.1. $U'(\cdot) > 0, U''(\cdot) < 0, U'''(\cdot) > 0$.

$V(0) = 0, V'(m) > 0, V''(m) < 0$.

$\lambda_i(x, e) \equiv c_i e^{-c_0 \times [r(x) + e]}$ for some constants c_i, c_0 , some function $r(x)$ and for $i \in S, O, M$.

Thus $\frac{-\partial \lambda_i(x, e)}{\partial e} = c_0 \times \lambda_i(x, e) > 0$ and $\frac{\partial^2 \lambda_i(x, e)}{\partial e^2} = c_0^2 \times \lambda_i(x, e) > 0$.

$\Psi'(e) > 0, \Psi''(e) \geq 0$.

2.2 Effect of aggregate mileage reduction on individual accident risk

Suppose under type- i pricing ($i \in O, M$) society reduces driving (relative to type-S) by a share of ρ_i , that is $1 - \rho_i = M_i/M_S$ where M_j denotes aggregate mileage under Type- j pricing, $j \in S, O, M$. Let λ_i denote the mean rate of driving incidents, provided the same driver exerts the same effort as before. Similarly, denote the type- i conditional probabilities of having a one- and two-car accident by γ_1^i and γ_2^i . The rates

of 1-car accidents and traffic violations are (assumed to be) independent of how many cars there are in the streets, whereas the rate of 2-car accidents should be reduced by a share of $\xi \times \rho_i$:

$$\begin{aligned}\lambda_S \times \gamma_1^S &= \lambda_i \times \gamma_1^i. \\ (1 - \xi \rho_i) \times \lambda_S \times \gamma_2^S &= \lambda_i \times \gamma_2^i. \\ \lambda_i \times (1 - \gamma_1^i - \gamma_2^i) &= \lambda_S \times (1 - \gamma_1^S - \gamma_2^S).\end{aligned}$$

Thereby ξ expresses the accident elasticity with respect to mileage. Parry (2005) argues that the appropriate value for ξ has not been well researched, despite the large potential benefits of programs like PAYD. Vickrey (1968) and Edlin (2003) estimate ξ at 1.5 and 1.7, but advise caution with these estimates.

Combining these three equations yields:

$$\begin{aligned}\lambda_i &= \lambda_S (1 - \gamma_2^S \xi \rho_i) \\ \gamma_1^i &= \frac{\gamma_1^S}{1 - \gamma_2^S \xi \rho_i} \\ \gamma_2^i &= \frac{\gamma_2^S (1 - \xi \rho_i)}{1 - \gamma_2^S \xi \rho_i} \\ 1 - \gamma_1^i - \gamma_2^i &= \frac{1 - \gamma_1^S - \gamma_2^S}{1 - \gamma_2^S \xi \rho_i}\end{aligned}\tag{1}$$

The average loss for a driving incident under standard pricing is:

$$L_S \equiv \gamma_1^S \frac{L}{2} + \gamma_2^S L,\tag{2}$$

and for type-i pricing:

$$\begin{aligned}L_i(\rho_i) &\equiv \gamma_1^i \frac{L}{2} + \gamma_2^i L \\ &= L \times \frac{\gamma_1^S + \gamma_2^S - \xi \rho_i \gamma_2^S}{1 - \gamma_2^S \xi \rho_i} \\ &= L_S \times \frac{\frac{\gamma_1^S}{2} + (1 - \xi \rho_i) \gamma_2^S}{(1 - \gamma_2^S \xi \rho_i) (\frac{\gamma_1^S}{2} + \gamma_2^S)}\end{aligned}\tag{3}$$

Note that the right hand side of equation (3) is less than L_S if and only if ρ_i is positive. Also: $\lambda_i < \lambda_S$, $\gamma_1^i > \gamma_1^S$, $\gamma_2^i < \gamma_2^S$ and $1 - \gamma_1^i - \gamma_2^i > 1 - \gamma_1^S - \gamma_2^S$ if and only if $\rho_i > 0$. In other words, as society reduces driving ($\rho_i > 0$), the frequency of driving incidents, the average loss from an incident and the conditional probability of having a 2-car accident go down, whereas the conditional probabilities of 1-car accidents and traffic violations increase (although the unconditional per-mile probabilities for the latter two stay the same), as we would expect.

2.3 Type S: Standard per-period premiums

A driver of type (x, θ_q) will then

$$\max_{\{m,e\}} E [U(Y - \Pi_S - gm) + V_x(m) - m\Psi(e)] \quad (4)$$

subject to

$$\Pi_S = \alpha q_S + (1 - \alpha)NL_S \quad (5)$$

$$N \sim \text{Poisson}(\lambda_S(x, e) m). \quad (6)$$

That is,

$$\max_{\{m,e\}} \sum_{n=0}^{\infty} \frac{e^{-\lambda_S m} (\lambda_S m)^n}{n!} U(Y - \alpha q_S - (1 - \alpha)nL_S - gm) + V_x(m) - m\Psi(e). \quad (7)$$

For simplicity, let's define

$$u_S(n) \equiv U(Y - \alpha q_S - (1 - \alpha)nL_S - gm) \quad (8)$$

$$u'_S(n) \equiv U'(Y - \alpha q_S - (1 - \alpha)nL_S - gm)$$

Note that

$$u_S(n+1) - u_S(n) \approx u'_S(n)(-L_S)(1 - \alpha). \quad (9)$$

This yields the first-order conditions (with respect to m and e):

$$[\lambda_S(x, e)L_S(1 - \alpha) + g] E[u'_S(N)] + \Psi(e) - V'_x(m) = 0 \quad (10)$$

and

$$\frac{-\partial \lambda_S(x, e)}{\partial e} L_S(1 - \alpha) E[u'_S(N)] - \Psi'(e) = 0. \quad (11)$$

And combining (10) and (11), we obtain

$$\frac{\Psi'(e)}{\frac{-\partial \lambda_S(x, e)}{\partial e}} \times \left[\lambda_S(x, e) + \frac{g}{(1 - \alpha)L_S} \right] - V'_x(m) + \Psi(e) = 0. \quad (12)$$

2.4 Type O: Per-mile pricing with odometer readings

A driver of type (x, θ_q) will then

$$\max_{\{m,e\}} E [U(Y - \pi_O m - gm) + V(m) - m\Psi(e)] \quad (13)$$

subject to

$$\pi_O = \alpha q_O + (1 - \alpha) \frac{NL_O}{m} \quad (14)$$

$$N \sim \text{Poisson}(\lambda_O(x, e) m). \quad (15)$$

That is,

$$\max_{\{m, e\}} \sum_{n=0}^{\infty} \frac{e^{-\lambda_O m} (\lambda_O m)^n}{n!} U(Y - \alpha q_O m - (1 - \alpha)nL_O - gm) + V_x(m) - m\Psi(e). \quad (16)$$

For simplicity, let's define

$$u_O(n) \equiv U(Y - \alpha q_O m - (1 - \alpha)nL_O - gm) \quad (17)$$

$$u'_O(n) \equiv U'(Y - \alpha q_O m - (1 - \alpha)nL_O - gm)$$

Note that

$$u_O(n+1) - u_O(n) \approx u'_O(n)(-L_O)(1 - \alpha). \quad (18)$$

This yields the first-order conditions (with respect to m and e):

$$[\lambda_O(x, e)L_O(1 - \alpha) + \alpha q_O + g] E[u'_O(N)] + \Psi(e) - V'_x(m) = 0 \quad (19)$$

and

$$\frac{-\partial \lambda_O(x, e)}{\partial e} L_O(1 - \alpha) E[u'_O(N)] - \Psi'(e) = 0. \quad (20)$$

And combining (19) and (20), we obtain

$$\frac{\Psi'(e)}{\frac{-\partial \lambda_O(x, e)}{\partial e}} \times \left[\lambda_O(x, e) + \frac{g}{(1 - \alpha)L_O} + \frac{\alpha q_O}{(1 - \alpha)L_O} \right] - V'_x(m) + \Psi(e) = 0. \quad (21)$$

2.5 Type M: Pricing under monitoring

If the insurer monitors style of driving he knows each driver's true per-mile accident risk $\lambda_M(x, e)$ where e is driver (x)'s effort in the current period. He can therefore apply the following rule to determine per-mile premiums (after observing her driving behavior over the past period):

$$\pi_M = \lambda_M(x, e) L_M. \quad (22)$$

In addition to the expected utility maximization the driver also faces a loss of κP for privacy valuations. Thereby κ represents the share of privacy concerns the government leaves on the table, in the sense that if the government increases privacy protection by imposing restrictions on the usage of data collected via EDRs, κ will fall below 1.

A driver of type (x, θ_q) will then

$$\max_{\{m,e\}} U(Y - \pi_M m - gm) + V_x(m) - m\Psi(e) - \kappa P \quad (23)$$

subject to (22).

That is,

$$\max_{\{m,e\}} U(Y - \lambda_M(x, e)L_M m - gm) + V_x(m) - m\Psi(e) - \kappa P. \quad (24)$$

For simplicity, let's define

$$u'_M() \equiv U'(Y - \lambda_M(x, e)L_M m - gm) \quad (25)$$

This yields the first-order conditions (with respect to m and e):

$$[\lambda_M(x, e)L_M + g] u'_M() + \Psi(e) - V'_x(m) = 0 \quad (26)$$

and

$$\frac{-\partial \lambda_M(x, e)}{\partial e} L_M u'_M() - \Psi'(e) = 0. \quad (27)$$

And combining (26) and (27) we obtain

$$\frac{\Psi'(e)}{\frac{-\partial \lambda_M(x, e)}{\partial e}} \times \left[\lambda_M(x, e) + \frac{g}{L_M} \right] - V'_x(m) + \Psi(e) = 0. \quad (28)$$

2.6 Discussion

Let us assume all three optimization problem have an interior solution as given by the respective First Order Conditions.

Lemma 2.1.

- (a) The left-hand-sides of equations (10), (19) and (26) are increasing in m and decreasing in e .
- (b) The left-hand-sides of equations (11), (20) and (27) are increasing in m and decreasing in e .
- (c) The left-hand-sides of equations (12), (21) and (28) are increasing in both m and e .

Proof. Parts (b) and (c) follow by virtue of Assumption 2.1. So does the first part of (a). To analyze (a) with respect to e , take the derivative of the LHS of (10) with respect to e . It works the same for the other FOC.

$$\begin{aligned} \frac{\partial(10)}{\partial e} &= \frac{-\partial \lambda_S(x, e)}{\partial e} L_S(1 - \alpha)E[u'_S(N)] + \lambda_S(x, e) [L_S(1 - \alpha) + g] \frac{\partial E[u'_S(N)]}{\partial e} + \Psi'(e) \\ &= \lambda_S(x, e) [L_S(1 - \alpha) + g] \frac{\partial E[u'_S(N)]}{\partial e}, \end{aligned}$$

where the last equation follows from applying (11). By Assumption 2.1, $\frac{\partial E[u'_S(N)]}{\partial e} < 0$, and thus $\frac{\partial(10)}{\partial e} < 0$. \square

Proposition 2.1. *For each type of insurance pricing and each driver, if an interior solution exists to their optimization problem and if this solution is described by the FOC, then this solution is unique.*

Proof. Follows from Lemma 2.1 by considering the FOC with respect to m and the combined FOC. \square

Moreover, any two of the three first-order-conditions in each section fully describe the driver's optimal choice of m and e , so that we can use any two of these equations to compare driving behavior under different types of insurance pricing.

2.6.1 Type S vs. Type O

Let's consider all drivers of observable characteristic $Q = \theta_q$, and focus on the driving decisions made by a driver of arbitrary type $X = x$. Suppose all other drivers of this cohort behave the same way under type S and type O. Thus $\rho_O = 0$, and therefore $L_O = L_S$, $\lambda_O(x, e) \equiv \lambda_S(x, e)$. Also under per-mile pricing with odometer readings, the per-mile premium q_O for this cohort will be in one-to-one correspondence with the per-period premium q_S under standard insurance pricing:

$$\int q_O m_O(x) dF_{X|Q}(x|\theta_q) = \int q_S dF_{X|Q}(x|\theta_q) = q_S. \quad (29)$$

That is,

$$q_O = \frac{q_S}{E[m_O|Q = \theta_q]} \quad (30)$$

In other words, q_O will be a fair premium to the *average* (mileage) driver. Note that this equation is accurate only when $\lambda_O = \lambda_S$.

First, compare (12) with (21). The LHS of (12) is smaller due to the (αq_O) -term which represents the additional guaranteed marginal cost (premium) under PAYD from driving an additional mile. This implies that a driver under odometer-based pricing would choose lower mileage and/or lower effort.

Second, note that (11) and (20) look rather similar, the sole difference being the arguments of the marginal utility functions. Which of these arguments is bigger depends on the driver's mileage. For a low-frequency driver, $q_O m < q_S$, so that $u'_S(N) > u'_O(N)$. To satisfy (20) the driver would need to increase mileage and/or reduce effort. In contrast, for a frequent driver $q_O m > q_S$ and thus $u'_S(N) < u'_O(N)$. This gives the driver incentives to reduce mileage and/or increase effort under PAYD. And finally, for a driver with an average number of miles, the arguments are roughly the same and the two equations are identical. To remain in equilibrium the driver would need to either increase or decrease both mileage and effort.

Combining these comparisons of first-order conditions leads to the following results:

Proposition 2.2.

Frequent drivers: $m_O < m_S$, *effort ambiguous*.

Average frequent drivers: $m_O < m_S$, $e_O < e_S$.

Infrequent drivers: *mileage ambiguous*, $e_O < e_S$.

□

The two sets of equations nicely point out the intuition behind these results: The overall mileage-reduction can be explained by the increased marginal cost of driving, as expressed in equations (12) and (21). However, there is more to the story. For one, this also leads to a reduction in caretaking. Consider both a reduction in driving as well as an increase in effort as means for the driver to reduce her accident risk. If increased marginal costs of driving make her decrease her mileage, she already has less accident risk and thus also less incentives to exert effort.

On the other hand, equations (11) and (20) express the income effect and its impact on mileage and accident prevention effort. Frequent drivers will see their expected overall premiums rise under per-mile pricing, which reduces composite-good consumption and in turn increases marginal utility thereof. PAYD thus provides them with more incentives to cut down insurance costs and save money that they can spend on other goods. This can be achieved under PAYD by a further reduction in driving, affecting both the basic premium and accident risk (in the form of a premium surcharge). The latter can also be reduced by an increase in caretaking, which as well is justified by the higher marginal benefits from avoiding an accident. For the most frequent driver this income effect might even offset and reverse her incentives to reduce effort as a result of the increase in marginal driving costs.

An infrequent driver however will pay a lot less for car insurance under usage-based pricing, so she has additional money that she can allocate among both driving and composite-good consumption. And for the most infrequent drivers (if at all) the explicit savings under the new pricing system will give her so much money (which she will partly allocate into driving) that it outweighs the increase in marginal driving costs so that she ends up driving more under per-mile pricing than before. As for caretaking efforts, while an accident would increase an infrequent drivers aggregate premium by the same amount as for a frequent driver, these costs affect her less because she has more money to spare due to lower overall premiums. In fact this effect will likely reduce her caretaking efforts even further, as compared to Type S pricing.

On aggregate, therefore, $M_O < M_S$, that is $\rho_O > 0$, $L_O < L_S$, $\lambda_O(x, e) < \lambda_S(x, e)$, $\frac{-\partial\lambda_O(x, e)}{\partial e} < \frac{-\partial\lambda_S(x, e)}{\partial e}$ ²⁴. This will reduce the LHS of equation (20) (thus putting upward pressure on m_O and downward pressure on e_O) and reduce the LHS of (19) (thus putting upward pressure on m_O). It also increases the LHS of (21) (which gives the driver incentives to reduce m_O and/or e_O). Altogether, part of the incentives to reduce driving will be undone by the fact that under usage-based pricing there will be fewer cars in the streets. Moreover, it will further decrease drivers' incentives to exert effort to prevent accidents for the very same reason.

In conclusion, the level of aggregate mileage reduction is determined not only by an increase in marginal driving costs under PAYD, but also by an income effect that will provide frequent drivers with more overall incentives to reduce mileage. This also makes the average driver on the road less skilled as compared to standard pricing, since frequent drivers tend to be better drivers as well. Combined with the induced changes in drivers' caretaking efforts, this might negatively impact the accident benefits from switching to a system of per-mile premiums in a way that is not accounted for by the current literature. Furthermore we see that

²⁴But still $\frac{\lambda_O(x, e)}{-\partial\lambda_O(x, e)} = \frac{\lambda_S(x, e)}{-\partial\lambda_S(x, e)}$.

drivers will respond to an overall reduction in mileage with more frequent and less careful driving which further reduces aggregate benefits.

2.6.2 Type O vs. Type M

Let us again consider all drivers of characteristic $Q = \theta_q$, and focus on the driving decisions made by a driver of arbitrary type $X = x$. Suppose all other drivers of this cohort behave the same way under Type O and Type M pricing. Thus $\rho_O = \rho_M$, and therefore $L_O = L_M$, $\lambda_O(x, e) \equiv \lambda_M(x, e)$.

First, compare (21) with (28). The latter is of course independent of α , and the two equations are identical for $\alpha = 0$, but the LHS of (21) is always larger for positive α . In fact, the difference is increasing in α (to infinity as α approaches 1). By Lemma 2.1 this indicates that a GPS device in a car puts an upward pressure on mileage and/or effort, and the pressure increases with α . For very small α however, these equations are roughly identical and higher mileage under one type would have to be "balanced out" by a choice of less effort as compared to the other pricing type.

Second, compare (20) with (27). Note first that since $U'''(\cdot) > 0$, U' is convex, thus by *Jensen's Inequality*:

$$E[U'(Z)] > U'(E[Z])$$

for any random variable Z . Thus:

$$\begin{aligned} E[u'_O(N)] &= E[U'(Y - \alpha q_O m - (1 - \alpha)NL_O - gm)] \\ &> U'(E[Y - \alpha q_O m - (1 - \alpha)NL_O - gm]) \text{ by Jensen's Inequality} \\ &= U'(Y - m[\alpha q_O + (1 - \alpha)\lambda_O L_O + g]). \end{aligned} \quad (31)$$

For α close to 0, $\alpha q_O + (1 - \alpha)\lambda_O L_O + g \approx \lambda_O L_O + g = \lambda_M L_M + g$ and therefore:

$$\begin{aligned} E[u'_O(N)] &> U'(Y - m[\alpha q_O + (1 - \alpha)\lambda_O L_O + g]) \\ &\approx U'(Y - m[\lambda_M L_M + g]) \\ &= u'_M(). \end{aligned}$$

Hence the LHS of (20) exceeds the LHS of (27) which – combined with the previous argument – implies $m_O < m_M$ and $e_O > e_M$ for low α .

As α increases, the driver's risk aversion becomes less of a factor as part of the premium is now unaffected by her driving behavior. In addition, as $\alpha \gg 0$, changes in driving behavior differ depend on the riskiness of a driver.

For a safe driver, $\lambda_O(x, e)L_O \ll q_O$, and thus:

$$Y - m[\alpha q_O + (1 - \alpha)\lambda_O L_O + g] < Y - m[\lambda_O L_O + g],$$

and due to the concavity of $U(\cdot)$:

$$\begin{aligned} U'(Y - m[\alpha q_O + (1 - \alpha)\lambda_O L_O + g]) &> U'(Y - m[\lambda_O L_O + g]) \\ &= U'(Y - m[\lambda_M L_M + g]) \\ &= u'_M(). \end{aligned}$$

On the other hand, for a risky driver, $\lambda_O(x, e)L_O \gg q_O$, and thus:

$$U'(Y - m[\alpha q_O + (1 - \alpha)\lambda_O L_O + g]) < u'_M().$$

That is, for safe drivers $E[u'_O(N)] > u'_M()$ while for risky drivers there is a countereffect that makes this inequality more ambiguous and potentially change signs.

In any event, for any driver $E[u'_O(N)] - u'_M()$ is bounded for $0 \leq \alpha \leq 1$. Thus as $\alpha \nearrow 1$ the LHS of (27) exceeds (20), and we can conclude that for all drivers there exists $0 < \alpha_0 < 1$ such that (27) $\dot{>}$ (20) if and only if $\alpha > \alpha_0$. This implies (applying Lemma 2.1) that for large enough α : $m_M < m_O$ and/or $e_M > e_O$, although the mileage effect is more prevalent with risky drivers. This is intuitive as they face the highest premiums under monitoring and an additional unit of effort goes a longer way than for a safe driver.

Finally, comparing (20) and 27 for $\alpha \gg 0$, for a safe driver (20) exceeds (27), which puts upward pressure on mileage and/or downward pressure on effort, under monitoring.

In combination with the previous results:

$$(19)/(26): m_M > m_O \text{ and/or } e_M < e_O,$$

$$(20)/(27): m_M < m_O \text{ and/or } e_M > e_O,$$

$$(21)/(28): m_M > m_O \text{ and/or } e_M > e_O,$$

we obtain (for a safe driver with $\alpha \gg 0$) $m_M > m_O$ and $e_M > e_O$ ²⁵.

For a risky driver however, $U'(Y - m[\alpha q_O + (1 - \alpha)\lambda_O L_O + g]) < u'_M()$, all the while $\alpha q_O + (1 - \alpha)\lambda_O L_O + g < \lambda_M(x, e)L_M + g$, and therefore we cannot say whether (19) exceeds (27) or vice versa. We thus have to rely on our previous comparisons of (20) vs. (27) and (20) vs. (27), as summarized in (2.6.2) to conclude that for a risky driver under $\alpha \gg 0$: $e_M > e_O$, while mileage is ambiguous and may not change much at all. In conclusion:

Proposition 2.3.

For $\alpha \approx 0$: $m_O < m_M$, $e_O > e_M$.

For $\alpha \gg 0$: $e_O < e_M$. Moreover, for a safe driver: $m_O < m_M$.

□

²⁵*Proof* (by contradiction):

Suppose $m_M \leq m_O$. Then by (21) vs. (28): $e_M > e_O$ and by (19) vs. (26): $e_M < e_O$. *Contradiction!*

Suppose $e_M \leq e_O$. Then by (21) vs. (28): $m_M > m_O$ and by (20) vs. (27): $m_M < m_O$. *Contradiction!*

□

The results in the first part are intuitively driven by a driver's risk aversion that makes her prefer the certain premium (λLm , which she pays when being monitored) over the lottery (NL , without monitoring). In that sense, monitoring provides some sort of "insurance" on a driver's (future) premium, and consequently the driver's behavior will be subject to moral hazard: she has less incentives to avoid accidents and thus drives more frequent and less careful.

As α increases, that is as Type O premiums depend less on the number of accidents, the insurance feature of monitoring is overshadowed by the reduction of influence a driver has on her premium under odometer-based pricing, which diminishes the benefits from exerting effort.

Moreover, for a given α safe drivers seem to be more likely to increase their miles driven ($m_M > m_O$), whereas the most risky drivers might even reduce driving under type-M pricing ($m_M < m_O$). This can be explained by risky drivers paying higher per-mile premiums under monitoring as compared to type-O pricing, so they have more incentives to reduce driving in order to cut expenses. Safe drivers on the other hand save money under monitoring and have thus additional funds available that they can partly spend on driving. Overall, there seems to be a trend that for any $\alpha \in [0, 1]$ aggregate mileage will increase under monitoring, that is $M_M > M_O$, as only the most risky drivers seem to have incentives to actually reduce their driving. An intuitive explanation is that insurance premiums can be reduced by lower mileage and higher effort. If drivers are given exogenous incentives to increase effort (in our case through monitoring), then they can drive more and still keep their premium at the same level.

3 A dynamic pricing model for Pay-As-You-Drive car insurance

3.1 Description of the model

I present a dynamic equilibrium pricing framework for auto insurance based on the aggregation of individual driving behavior. The model allows to numerically quantify the differences in driving behavior – individual and aggregate – between various types of insurance pricing: Primarily types S, O and M, as described in section 2.1, where only a single pricing scheme is available to drivers. However the model can be extended to analyze driving behavior and market equilibria when drivers have the choice between several pricing schemes.

This framework is a dynamic extension of the model in the previous section and inherits most of its features (see sections 2.1 and 2.2). The only difference is that in this dynamic setting the (non-monitoring) insurer can keep track of driving records and make premiums actively depend on it. Conceptually, this difference comes down to whether having an accident will cost the driver at the end of that period only (as in section 2) or whether the costs are distributed over all future periods in the form of a premium adjustment (as in this section).

3.2 Type S: Standard per-period premiums

Consider the cohort of drivers with basic characteristics θ_q . For this cohort, let a driver's insurance premium be determined as follows:

$$\Pi_t = \alpha \times q + (1 - \alpha) \times \Pi_t^{DR} \quad (32)$$

where

$$\Pi_{t+1}^{DR} = \beta \times N_t L + (1 - \beta) \times \Pi_t^{DR}. \quad (33)$$

In principle you can think of the premium Π_t as a linear combination (with weight α) of the estimated accident risk based on basic characteristics, q , and the estimated accident risk based on driving record, Π_t^{DR} . Note that q is the same for all drivers of this cohort and is time-invariant, while the individual- and time specific value for Π_t^{DR} is determined by equation (33). The appropriate value of q will be such that the competitive insurer expects to break even with this cohort, provided that all drivers choose mileage and effort optimally (that is to maximize their own expected utility). Another way of thinking about this formula is that q represents the average premium by drivers of this cohort, and the actual premium will result from a deviation up or down depending on driving record. β refers to the weight the insurer puts on recent incidents relative to past driving history. For instance, $\beta = 0.2$ corresponds to a half-life of recent information of $\frac{\ln(1/2)}{\ln(1-0.2)} \approx 3.1$ periods.

Combining (32) and (33), we obtain:

$$\begin{aligned} \Pi_{t+1} &= \alpha \times q + (1 - \alpha) \times [\beta \times N_t L_S + (1 - \beta) \times \Pi_t^{DR}] \\ &= (1 - \beta) \times \Pi_t + \beta \times [\alpha \times q + (1 - \alpha) \times N_t L_S] \end{aligned} \quad (34)$$

Note that equation (34) indicates that to compute the next period premium only the current premium, the number of driving violations in this period as well as knowledge of q , α and β are required. In other words, under the present setup, all necessary information on the driver's accident history is included in Π_t .

3.2.1 Individual maximization

A driver of type (x, θ_q) will then

$$\max_{\{m_t, e_t\}_{t=1}^{\infty}} E \left[\sum_{t=1}^{\infty} \delta^{t-1} [U(Y_t - \Pi_t - g_t m_t) + V(m_t) - m_t \Psi(e_t)] \right] \quad (35)$$

subject to (34) and

$$N_t \sim \text{Poisson}(\lambda_S(x, e_t) m_t). \quad (36)$$

Assuming Y and g are non-stochastic and constant over time, this yields the Bellman Equation

$$J_S(\Pi) = \max_{\{m, e\}} U(Y - \Pi_S - g m) + V(m) - m \Psi(e) + \delta E J_S(\Pi'_S) \quad (37)$$

subject to

$$\Pi'_S = (1 - \beta) \times \Pi_S + \beta \times [\alpha \times q_S + (1 - \alpha) \times NL_S] \quad (38)$$

$$N \sim \text{Poisson}(\lambda_S(x, e) m), \quad (39)$$

where $J_S(\cdot)$ is the value function for standard insurance pricing in terms of per-period premium Π_S .

We can rewrite the Bellman equation (37):

$$J_S(\Pi) = \max_{\{m, e\}} U(Y - \Pi_S - gm) + V(m) - m\Psi(e) + \delta \sum_{n=0}^{\infty} \frac{e^{-\lambda_S m} (\lambda_S m)^n}{n!} J_S(h_S(\Pi_S, n)) \quad (40)$$

subject to

$$h_S(\Pi_S, n) = (1 - \beta) \times \Pi_S + \beta \times [\alpha \times q_S + (1 - \alpha) \times nL_S] \quad (41)$$

$$\lambda_S = \lambda_S(x, e).$$

The first-order conditions (with respect to e and m) are:

$$\begin{aligned} \Psi'(e) &= \delta \frac{\partial \lambda_S(x, e)}{\partial e} \sum_{n=0}^{\infty} \frac{e^{-\lambda_S(x, e)m} (\lambda_S(x, e)m)^n}{n!} \times [J_S(h_S(\Pi_S, n+1)) - J_S(h_S(\Pi_S, n))] \\ &= \delta \frac{\partial \lambda_S(x, e)}{\partial e} E [J_S(h_S(\Pi_S, N+1)) - J_S(h_S(\Pi_S, N))] \end{aligned} \quad (42)$$

and

$$U'(Y - \Pi - gm)g + \Psi(e) - V'(m) = \delta \lambda_S(x, e) E [J_S(h_S(\Pi_S, N+1)) - J_S(h_S(\Pi_S, N))] \quad (43)$$

subject to (41) and (39).

Combining the two first-order conditions, we can get rid of the sum and obtain

$$U'(Y - \Pi - gm)g + \Psi(e) - V'(m) = \frac{\lambda_S}{\frac{\partial \lambda_S(x, e)}{\partial e}} \Psi'(e). \quad (44)$$

Note that $E [J_S(h_S(\Pi_S, N+1)) - J_S(h_S(\Pi_S, N))] \approx \beta(1 - \alpha)L_S E [J'_S(h_S(\Pi_S, N))]$.

Given q we can solve these equations numerically for optimal mileage $m = m_x^*(\Pi|q)$ and effort $e = e_x^*(\Pi|q)$ as driver (x, θ_q) 's best response to any insurance premium under the standard pricing system.

3.2.2 Aggregation of drivers

Our goal is to find a stable ("Markovian", that is time-invariant) equilibrium in the insurance market under the respective pricing system. To accomplish that we require every type of driver (x, θ_q) to be in equilibrium individually, in the sense that her probability distribution over the premium she faces remains the same over

time. Let's consider all drivers of type (x, θ_q) , and let a value for q be set by the insurer²⁶.

Denote the beginning-of-period distribution (the pdf to be precise) of the driver type in per-period premiums by $d_x(\Pi|q)$. Let all drivers choose mileage and effort optimally as determined in the previous subsection. The end-of-period distribution will then be

$$d'_x(\Pi|q) = \sum_{n=0}^{\infty} \frac{e^{-\lambda_S(x, e_x^*(\tilde{\Pi}(n)|q))} m_x^*(\tilde{\Pi}(n)|q) (\lambda_S(x, e_x^*(\tilde{\Pi}(n)|q)) m_x^*(\tilde{\Pi}(n)|q))^n}{n!} \times d_x(\tilde{\Pi}(n)|q) \quad (45)$$

where $\tilde{\Pi}(n) = \frac{\Pi - \beta[\alpha q + (1-\alpha)nL_S]}{1-\beta}$ as described in equation (38).

To guarantee equilibrium, this distribution needs to be stationary, that is $d_x(\cdot|q) \equiv d'_x(\cdot|q)$. Let us denote the stationary pdf by $d_x^*(\Pi|q)$ and corresponding cdf by $D_x^*(\Pi|q)$. Let us suppose that for every type of driver we have found such a stationary distribution $d_x^*(\Pi|q)$.

The aggregate pdf (whereby we mean aggregate over x for all drivers with characteristics θ_q) which we denote by $\bar{d}_{\theta_q}^*(\Pi|q)$ is thus stationary as well and can be computed via

$$\bar{d}_{\theta_q}^*(\Pi|q) = \int d_x^*(\Pi|q) dF_{X|Q}(x|\theta_q). \quad (46)$$

Denote the corresponding cumulative distribution function by $\bar{D}_{\theta_q}^*(\Pi|q)$.

We can also compute the aggregate loss among all drivers with characteristics θ_q :

$$\bar{L}_s(q) = \int \int \lambda(x, e_x^*(\Pi|q)) m_x^*(\Pi|q) L_S dD_x^*(\Pi|q) dF_{X|Q}(x|\theta_q) \quad (47)$$

as well as aggregate premiums collected (in the current period) for this cohort:

$$\bar{\Pi}_s(q) = \int \Pi dD_{\theta_q}^*(\Pi|q). \quad (48)$$

If these two equations resulted in the same number, we would have found the optimal q that makes premiums collected equal to expected losses so that the insurer breaks even. Since this is generally not the case, we need to find some way to iterate over q . Increasing q implies higher premiums for all drivers which makes aggregate premiums go up. It also gives drivers a bigger chance to reduce premiums by avoiding accidents, so they have incentives to increase effort and reduce mileage. That is aggregate losses should go down. That is: if $\bar{L}_s(q) > \bar{\Pi}_s(q)$ we need to increase q and vice versa²⁷. We can thus iterate over q to find optimal

²⁶Note that this implies that if the distribution is the same for two consecutive periods, it is the same in the following period as well and is thus stationary, since the only variable that affects driving choices and future premiums is Π .

²⁷If the insurer breaks even this period, he is also expected to break even in the next period: Aggregate premiums for next period can be written as

$$\bar{\Pi}'_s(q) = \int \int \Pi' dD_x^*(\Pi|q) dF_{X|Q}(x|\theta_q)$$

where Π' is given in equation 38. Therefore

$$\begin{aligned} E\bar{\Pi}'_s(q) &= \int \int (1-\beta) \times \pi + \beta \times [\alpha \times q + (1-\alpha) \times \lambda_S(x, e_x^*(\Pi|q)) m_x^*(\Pi|q) L_S] dD_x^*(\Pi|q) dF_{X|Q}(x|\theta_q) \\ &= (1-\beta)\bar{\Pi}_s(q) + \beta[\alpha q + (1-\alpha)\bar{L}_s(q)]. \end{aligned}$$

$q^*(\theta_q)$.

For future reference, the aggregate mileage under standard insurance pricing is:

$$M_S \equiv \int \int \int m_x^*(\Pi_S | q_S^*(\theta_q)) dD_x^*(\Pi_S | q_S^*(\theta_q)) dF_{X|Q}(x|\theta_q) dF_Q(\theta_q) \quad (49)$$

3.3 Type O: Per-mile pricing with odometer readings

Given the insurer's lack of information regarding true accident probabilities, the case of the insurer charging per-mile premiums without monitoring can be treated similarly to the standard type of auto insurance pricing, as analyzed in section (3.2). Denoting by q_O the per-mile accident risk if purely based on basic characteristics, and applying the same rule to determine premiums as with standard pricing, we obtain (similar to equation (34))

$$\pi_{t+1} = (1 - \beta) \times \pi_t + \beta \times [\alpha \times q_O + (1 - \alpha) \times \frac{N_t L_O}{m_t}] \quad (50)$$

3.3.1 Individual maximization

This is very similar to the case of standard pricing, section (3.2.1). A driver of type (x, θ_q) will choose to

$$\max_{\{m_t, e_t\}_{t=1}^{\infty}} E \left[\sum_{t=1}^{\infty} \delta^{t-1} [U(Y_t - \pi_t m_t - g_t m_t) + V(m_t) - m_t \Psi(e_t)] \right] \quad (51)$$

subject to (50) and

$$N_t \sim \text{Poisson}(\lambda_O(x, e_t) m_t). \quad (52)$$

Assuming Y and g are non-stochastic and constant over time, this yields the Bellman Equation

$$J_O(\pi_O) = \max_{\{m, e\}} U(Y - \pi_O m - gm) + V(m) - m \Psi(e) + \delta E J_O(\pi'_O) \quad (53)$$

subject to

$$\pi'_O = (1 - \beta) \times \pi_O + \beta \times \left[\alpha \times q_O + (1 - \alpha) \times \frac{N L_O}{m} \right] \quad (54)$$

$$N \sim \text{Poisson}(\lambda_O(x, e) m), \quad (55)$$

where $J_O(\cdot)$ is the value function for type-O pricing in terms of per-mile premium π_O .

We can thus rewrite the Bellman Equation (53):

$$J_O(\pi_O) = \max_{\{m, e\}} U(Y - \pi_O m - gm) + V(m) - m \Psi(e) + \delta \sum_{n=0}^{\infty} \frac{e^{-\lambda_O m} (\lambda_O m)^n}{n!} J_O(h_O(\pi_O, m, n)) \quad (56)$$

Since for the optimal $q = q^*$ we will also have $E \bar{\Pi}'_s(q^*) = \bar{\Pi}_s(q^*)$, we can conclude that

$$q^* = \bar{\Pi}_s(q^*) = \bar{L}_s(q^*)$$

, that is the optimal q in fact corresponds to the aggregate (that is average) premium collected from the cohort θ_q .

subject to

$$\begin{aligned} h_O(\pi, m, n) &= (1 - \beta) \times \pi_O + \beta \times \left[\alpha \times q_O + (1 - \alpha) \times \frac{nL_O}{m} \right] \\ \lambda_O &= \lambda_O(x, e). \end{aligned} \quad (57)$$

The first-order conditions (with respect to e and m) are:

$$\Psi'(e) = \delta \frac{\partial \lambda_O(x, e)}{\partial e} E [J_O(h_O(\pi_O, m, N + 1)) - J_O(h_O(\pi_O, m, N))] \quad (58)$$

and

$$\begin{aligned} U'(Y - \pi_O m - gm)(\pi + g) + \Psi(e) - V'(m) &= \delta E [-\lambda_O(x, e) J_O(h_O(\pi_O, m, N)) + \\ &\frac{1}{m} J_O(h_O(\pi_O, m, N + 1)) - \beta(1 - \alpha) \frac{nL_O}{m^2} J'_O(h_O(\pi_O, m, N))] \end{aligned} \quad (59)$$

Note again that $\beta(1 - \alpha)L_O E [J'_O(h_O(\pi_O, N))] \approx E [J_O(h_O(\pi_O, N + 1)) - J_O(h_O(\pi_O, N))]$.

Given q_O , L_O and $\lambda_O(\cdot, \cdot)$ we are able to solve these equations numerically for optimal mileage $m_O^*(\pi_O)$ and effort $e_O^*(\pi_O)$ as driver (x)'s best response to any insurance premium under the type-O pricing system. Since all uncertainty in regards to L_O and $\lambda_O(\cdot, \cdot)$ comes from the change in aggregate mileage (relative to the standard case) and can be expressed in a single variable, ρ_O , we only need to iterate over two variables, ρ_O and q_O .

3.3.2 Aggregation of drivers

Aggregating drivers under type-O pricing is very similar to the standard case, as described in section 3.2.2. The main difference is that now the iteration is over both q_O and ρ_O .

3.4 Type M: Pricing under monitoring

Assuming that when drivers are monitored premiums will be charged ex post, we obtain the same results as in the one-period model.

3.4.1 Individual maximization

A driver of type (x)²⁸ will choose to

$$\max_{\{m_t, e_t\}_{t=1}^{\infty}} E \left[\sum_{t=1}^{\infty} \delta^{t-1} [U(Y_t - \lambda_M L_M m_t - g_t m_t) + V(m_t) - m_t \Psi(e_t) - \kappa P] \right]. \quad (60)$$

²⁸Note that there is no need to include θ_q in the description of the driver as θ_q does not contain any additional information to the monitoring insurer, so that drivers will not be categorized based on their age etc.

Assuming again that Y and g are deterministic and constant over time, future premiums are non-stochastic from the driver's point of view, and we can see that every period the driver faces the same problem:

$$\max_{\{m,e\}} U(Y - \lambda_M L_M m - gm) + V(m) - m\Psi(e) - \kappa P, \quad (61)$$

which leads again to the First-Order-Conditions (26) and (27).

3.4.2 Aggregation of drivers

By the previous results, premiums in the case of a monitoring insurer are actuarially fair, so there is no need for an iteration to properly adjust parts of the premium (like the q in case of a non-monitoring insurer). The only unknowns in our model are L_M as given in equation (3) and $\lambda_M(\cdot, \cdot)$ which is proportional to $\lambda_S(\cdot, \cdot)$, with the coefficient being given in equation (1). Both are fully determined once we know the reduction in aggregate mileage, ρ_M , which we can compute by iterating over

$$1 - \rho_M = \frac{M_M(\rho)}{M_S}, \quad (62)$$

where

$$M_S(\rho_M) = \int \int m_x^*(\rho_M) dF_{X|Q}(x|\theta_q) dF_Q(\theta_q) \quad (63)$$

and M_S is given by equation (49). We can then solve numerically for optimal mileage m_x^* and effort e_x^* in the case of a monitoring insurer by iterating over ρ_M .

4 Appendix

4.1 Current implementations of Pay-As-You-Drive in practice

Over the past decade, more and more companies around the globe have actually adopted usage-based auto insurance, although mostly on a test basis and with very restricted availability to the general public.

Major U.S. auto insurer *Progressive Casualty Insurance Company* has been on the forefront of Pay-As-You-Drive ever since. In the late 1990s, Progressive successfully filed patents²⁹ to restrict other insurers from using a "method and system of determining a cost of automobile insurance based upon monitoring, recording and communicating data representative of operator and vehicle driving characteristics." In 1999, Progressive started its first usage-based insurance product, *Autograph* in the state of Texas. This was followed by *TripSense* in the states of Minnesota, Michigan and Oregon in 2004. In June 2008, Progressive launched *MyRate*, a voluntary program that allows drivers who wish to participate to install a chip in their car that records when, how much and how they drive, and to semiannually transfer the data to the insurer, who then provides a discount of up to 60%³⁰ off the per-period insurance premium, depending on the evaluated driving behavior. As of June 2009, MyRate is available in Alabama, Colorado, Connecticut, Georgia,

²⁹U.S. Patent Nos 5,797,134 and 6,064,970

³⁰<http://newsroom.progressive.com/2008/June/myrate-launch.aspx>, accessed on 06/28/2009

Kansas, Kentucky, Louisiana, Maryland, Michigan, Minnesota, Missouri, New Jersey, Oregon, Rhode Island and Texas³¹. Progressive states that the MyRate device does not record the location of driving, and claims that your premium is most likely to go down if you drive defensively, less than 10000 miles per year and rarely after midnight. In some states there is a \$30 charge per premium period for the device and the online transmission of data. Progressive also notes that MyRate data may be useful in determining the cause of an automobile accident, and that there may be legal obligations to release this data.

In the state of Washington, insurance company *Unigard* in corporation with the Washington State Department of Transportation is running a federally funded pilot project on usage-based insurance. The program recruits 5,000 vehicle owners across the state and is phased in over a period of five years. The data collected is intended to obtain the support of actuaries and regulators for PAYD³². This makes academic research on the pricing of per-mile auto insurance products all the more relevant.

In 2001, the Texas House passed Texas HB 45, the cents-per-mile choice law, authorizing insurance companies to charge car insurance premiums by the mile. Texas was the first state to change its insurance laws; others are now considering similar changes.

MileMeter Insurance Company, a fully licensed insurance carrier regulated by the Texas Department of Insurance, was the first company in the U.S. to offer an insurance-by-the-mile program³³. Drivers purchase miles in advance and are insured until they run out of miles. *MileMeter* does not require an on-board electronic device but receives odometer readings from the annual state inspection as well as from vehicle registration and private maintenance sources.

GMAC insurance offers *On Star* subscribers to save up to 54% on premiums if they allow GMAC Insurance to track their odometer readings through the telematics service³⁴. Reportedly, about 10,000 owners of GM vehicles have participated in a pilot program for the low-mileage discount since 2004.

PAYD car insurance products are offered outside the United States as well. For instance in Australia by *Real Insurance*, part of the multinational *Hollard Insurance Group*³⁵, and in Japan by *AIOI* in corporation with Toyota³⁶. *IRIS (International Research and Intelligent Systems) Global* is a software company based at the University of Warwick Science Park in Coventry, U.K., whose Pay-As-You-Drive motor insurance system helped them win *Strategic Risk* magazine's prestigious "European Risk Management Product of the Year 2008" award. The company's innovative PAYD product has recently undergone extensive trials with two insurers. Other major insurers and fleet management companies are now showing significant interest³⁷. Meanwhile in the U.K. *Norwich Union*, a frontrunner in introducing PAYD to the British market,

³¹<http://www.progressive.com/myrate/myrate-default.aspx>, accessed on 06/28/2009

³²<http://www.metrokc.gov/exec/news/2007/0327payasdrive.aspx>, accessed on 06/28/2009

³³<http://www.milemeter.com/about>, accessed on 06/28/2009

³⁴<http://www.edmunds.com/insideline/do/News/articleId=121750>, accessed on 06/28/2009

³⁵<http://www.payasyoudrive.com.au>, accessed on 06/28/2009

³⁶http://www.ioi-sonpo.co.jp/en/ar_pdf/ar2005e_01.pdf, page 37, accessed on 06/28/2009

³⁷<http://www.birminghampost.net/birmingham-business/birmingham-business-news/2008/05/>

has stopped offering these policies to private drivers, blaming the failure of carmakers to install the systems at the manufacturing stage³⁸.

4.2 Common arguments against Pay-As-You-Drive car insurance

Despite widespread agreement on the benefits of PAYD, some remain sceptic. Critiques have raised concerns with per-mile pricing on several fronts. While I do not believe that their arguments are strong enough to stand in the way of the revolutionary insurance product, some of them might explain why PAYD has not yet replaced standard auto insurance pricing, and why its implementation is taking so long.

Edlin (2003) argues that *monitoring costs* could substantially reduce individual savings. However, as PAYD becomes standard, both odometer readings and event data recorders should become significantly cheaper. Considering the substantial benefits that are attached to PAYD, the government could subsidize monitoring costs which would convince more drivers to switch. *Odometer* or *EDR fraud* should not be much of a concern since it is quite complicated and because the simple fact that it is already a crime will be enough to restrain most people from tampering.

Since individuals can't fully anticipate how much they drive, a *risk aversion* argument can be made in the sense that drivers prefer fixed over random premiums. However, gasoline is sold per gallon as well instead of a flat rate.

Lower price elasticity of driving: Some criticize per-mile premiums, arguing that drivers will not respond to price changes all that much. But whether drivers are really less price sensitive than commonly assumed can be tested with the data that is currently collected by several insurers worldwide.

Edlin (2003) also notes that *adverse selection* may be a concern for insurers: If per-mile premiums for a given pool of drivers are computed as the ratio of current annual premiums and average annual mileage, frequent drivers will leave the insurer towards a competitor charging standard insurance, and the insurer is stuck with infrequent drivers who tend to be more risky than average. For the remaining group of drivers, initial per-mile premiums are thus too low and the insurer will lose money. And even if all drivers are required to pay per-mile premiums, the frequent drivers will reduce their driving more than the rare drivers, so that the new per-mile risk under PAYD is again higher than the premium. This certainly poses a concern to regulators and actuaries alike. In section 3 I introduce a pricing model to compute equilibrium per-mile premiums that takes these effects into account.

Current laws and *regulation* may prevent the implementation of per-mile pricing in various states. I will give an overview of current regulatory positions in more detail in section 4.3.

The high level of regulation in the insurance industry may pose barriers for new entrants (who would typically be more likely to adopt a product as innovative as PAYD) [Bordoff and Noel (2008)].

Privacy advocates raise concern with insurers obtaining data on drivers via event data recorders. These *privacy concerns* will be explored in more detail in section 4.4.

A main reason why insurers have been rather unwilling to offer PAYD products is that they cannot internalize most of the externality gains they cause. If a driver switches to per-mile pricing and consequently reduces

12/iris-earns-top-european-accolade-65233-20899381/, accessed on 06/28/2009

³⁸http://www.companycardriver.co.uk/news/article/?art_ID=455143201

her driving, the reduction in accident risk for everybody else gets back to the insurer only to the extent that he covers the market. Not to mention the other externalities that will not benefit any insurer. However, as PAYD is becoming standard and available to the general public, more and more insurers will offer per-mile pricing in order to avoid losing customers. Then the insurer will keep sharing the externality benefits of his drivers with others, but he will also gain from his drivers having lower accident probabilities due to other insurers' drivers.

For the past decade, auto insurer *Progressive* has actively sought *patents* around innovations in telematic auto insurance. However, some experts doubt that the patents will hold up in court [Bordoff and Noel (2008)]. Moreover, *Progressive* states that it has licensed its patented usage-based insurance methods and systems to other companies in the past.

Prospective nature of insurance: Lastly, Edlin (2003) argues that insurers do not "want to collect ex post surcharges from drivers who drive a lot, particularly if the driver had no accident". Edlin adds that this problem could be surmounted easily if the insurer is working with a credit card company.

4.3 Legal and regulatory concerns surrounding PAYD

Guensler et al (2002) survey 43 State Insurance Commissioners regarding the potential barriers to implementing PAYD. They find that in many states PAYD programs can already be implemented, meaning that state laws and regulations do not specifically prohibit per-mile auto insurance pricing. Those states are: AZ, AL, CO, FL, GA, ID, IL, IA, KS, KY, MD, ME, MI, NV, OH, OK, OR, PA, SC, SD, TN, TX, UT, VA, WA, WV and WI.

However in most of these states there are a few minor regulatory barriers that not all PAYD programs might comply with. In Georgia, for instance, companies would need to ensure that their policy requires a down payment for sixty days of coverage and that the minimum insurance term is at least six months. Many states raise concerns that an appropriate PAYD policy would need to guarantee that drivers are insured at all times, which would not be true in the case of drivers pre-paying for miles. Over 90% of responding states require actuarial data proving that the new pricing structure is fair and equitable, and to be considered as such, New Mexico, for example, requires at least three years of data.

Only 40% of (25) respondents stated that they would allow for per-mile premiums simply based on the driver's current annual fees and reported mileage.

North Dakota specifically prohibits insurance companies from using EDR data (that is data obtained through electronic monitoring) to set insurance rates [McDonald and Cranor (2006)].

4.4 Privacy concerns when driving is monitored

As mentioned above, the collection of driving data with respect to how, where and when a car is driven raises privacy concerns. The data could be used to reconstruct an accident, potentially to the disadvantage of the driver. The insurer might come to the wrong conclusions when observing that you parked next to a bar for a few hours in the evening and later drive home. Or a third party that gains access to the data might infer from your driving patterns where you live and what time of the day you are not at home. To name just

a few controversial circumstances...

As of May 2005, roughly 25 million cars in the U.S. were equipped with an Event Data Recorder (EDR), with most drivers being unaware that they have one in their car [McDonald and Cranor (2006)].

In my economic analysis I treat locational privacy as a good that has a certain value (P) to drivers, and I assume they give up a share κ of it by having an EDR installed in their car. κ would equal 1 in the absence of governmental regulation that prohibits the use of monitoring data by either the insurer or a third party. Since privacy concerns are a serious social topic I believe that the widespread adoption of EDR-based auto insurance premiums will be accompanied by regulation to protect individual privacy. In that case we could reduce κ below 1. Troncoso et al (2007) propose the use of an alternative, privacy friendly, event data recorder that evaluates all data on board and only sends aggregate premium information to the insurer, without leaking locational information. Under this idea (which the authors call "PriPAYD") all rational privacy concerns would be eliminated and we obtain $\kappa = 0$ in the model.

The model allows for agents to be heterogeneous with respect to privacy valuation. This heterogeneity is reflected in a consumer survey, in which Cvrcek et al (2006) investigate the value of locational privacy in several European countries. The authors find that a majority of drivers name a value below 100 Euros, while some individuals would ask for more than 500 Euros for them to give up their locational privacy.

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