

# Capital Structure under Heterogeneous Beliefs and Agency Conflicts

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## Abstract

This paper investigates the effects of heterogeneous beliefs about future firm earnings and agency conflicts on corporate capital structure. We develop a dynamic structural model in which the manager of a firm has discretion in financing and effort and receives dynamic incentives through an explicit contract with shareholders in the presence of heterogeneous beliefs between the two parties. The manager's optimal contract implemented through financial securities leads to the firm's dynamic, incentive-driven capital structure. Accordingly, the theoretical and numerical analyses of the model generate several implications on the firm's capital structure: (i) Long-term debt declines with managerial optimism, while short-term debt increases with it; (ii) Long-term debt (Short-term debt) decreases (increases) with transient earnings risk, but increases (decreases) with intrinsic earnings risk; (iii) Equity value is negatively affected by managerial optimism, but positively affected by intrinsic earnings risk. Our analysis suggests that managerial optimism and transitory and permanent risk factors are important determinants of firms' optimal financial policies.

**JEL Classification:** G32

**Key Words:** Capital Structure, Imperfect Information, Heterogeneous Beliefs, Agency Conflicts, Managerial Ownership, Initial Public Offerings

# 1 Introduction

An individual's unrealistic optimism is widespread, and its degree seems to be greater if the person believes himself or herself to be in control, like an entrepreneur starting a new business (De Meza and Southey, 1996). While the impact of optimism at the individual decision-making level has been much discussed in the literature of behavioral economics, its effects at the aggregate level, such as firm level decisions, have not yet been completely explored. In particular, in corporate finance applications, in which differences in beliefs between firm managers (or insiders) and (outside) investors have long been assumed to be attributed to asymmetric information between the rational agents, relatively few studies have attempted to address the implications of heterogeneous prior beliefs such as entrepreneur optimism. This paper contributes to the little known literature by providing quantitative assessments of the impact of heterogeneous priors on corporate capital structure through a dynamic structural approach.

We briefly review a few studies that incorporate such heterogeneous prior beliefs into capital structure theory. First, some studies, with no consideration of agency conflicts, examine how heterogeneous beliefs affect a firm's choice between equity and debt financing. Despite different model settings, these studies all show a relatively optimistic manager's (compared to investors) tendency to use debt financing. From the psychological perspective that there are a dominant number of unrealistic optimists in the entrepreneurial class, De Meza and Southey (1996) derive the reliance of bank loans rather than equity finance as one of the stylized features of small-sized businesses. In a three-period model, Dittmar and Thakor (2007) analyze the financing choice of a manager who acts in the interests of shareholders even though they may object to the manager's investment decision in a project only because of their different beliefs about the value of the project. The analysis shows that compared to debt financing, in which debt-holders have differing interests and beliefs from the manager's, but which gives the benefit of tax shields, equity financing is more likely to be chosen by the manager when the agreement about the project's prospect between the manager and investors is higher. This result is also empirically demonstrated in their paper with several agreement parameters including the difference between a firm's actual EPS and the mean analyst forecast for the EPS. In contrast to the above studies, Chemmanur, Nandy, and Yan (2007) consider the optimism of outside investors and show that the firm will choose to issue equity if outsiders are more optimistic than management.

Another line of research deals with the effects of opinion difference between management and

investors in a setting of agency conflicts. In a two-period debt contracting framework of Landier and Thesmar (2008), entrepreneurs are allowed to choose at their own discretion between the growth and safe strategies based on the observed interim cash flow. It is derived in a unique competitive security equilibrium that optimists choose short-term debt contracts, whereas realists opt for long-term debt contracts, which is also tested using a data set of French entrepreneurs. Bigus (2003) compares standard financial contracts - pure debt, pure equity, and debt-equity mix - in a static model with heterogeneous beliefs and moral hazard problem (a risk averse manager's perk consumption problem) and argues that a debt-equity mix might outperform both pure debt and pure equity in terms of social surplus. Finally, Hackbarth (2004) shows that in a continuous-time capital structure model with managers' incentive to divert discretionary funds, managerial optimism and overconfidence can reduce the underinvestment problem through a higher debt level.

Our study complements these related studies in several respects. First, with the importance of agency conflicts as one of the key determinants of capital structure in mind, we develop a more concrete and realistic setting of agency conflicts in which a firm's undiversified manager and well-diversified shareholders (outside investors) dynamically interact with each other by providing actions that affect the firm's earnings. Second, unlike the above studies, we also consider heterogeneous posterior beliefs to be rationally updated over time from heterogeneous prior beliefs, which include both manager optimism and pessimism relative to investors. Third, we do not restrict the firm's financing contractual space, as opposed to some studies, and observe the firm's dynamic capital structure consisting of inside and outside equity, long-term debt, and short-term debt (cash reserve). Lastly, we calibrate our structural model and obtain quantitative insights into the level and effects of the heterogeneity in beliefs and those of earnings risks. In particular, by matching the model with different data sets, we can explain differences in financing policy of firms with differing levels of heterogeneity in beliefs and of earnings risk factors.

To that end, our model in the paper is based on the following two studies: one is Bhagat, Bolton, and Subramanian (2009), which provides a comprehensive capital structure model integrating both the trade-off theory and the agency theory on capital structure, and the other is Giat, Hackman, and Subramanian (2009), which develops a comprehensive dynamic principal-agent model with imperfect public information and asymmetric beliefs in which the principal and the agent both take value-enhancing actions. Following the former, we develop a dynamic setting in which a risk-averse manager determines both financing a new project and contracting with risk-neutral investors, and the manager's compensation structure from the contract can be derived by financial securities,

thereby resulting in the firm's dynamic capital structure. In order to model heterogeneous prior beliefs between the manager and shareholders and two distinctive risk factors of the firm's earnings, we rely on the latter study, specifically, the incremental termination payoff of the project, Bayesian learning process from heterogenous priors about the core output.

After theoretically and numerically deriving the equilibrium on the manager's optimal agency contract and optimal capital structure decision in the unified model, we provide the following capital structure implications. To begin with, the heterogeneity in beliefs does affect corporate capital structure: long-term debt and equity values decline with the degree of heterogeneity, while short-term debt increases with its degree. Furthermore, transitory and permanent risk factors of the firm's earnings have contrasting effects on its financing policies: (i) long-term debt decreases with transitory earnings risk, but increases with permanent earnings risk; (ii) short-term debt increases with transitory earnings risk, but decreases with permanent earnings risk; (iii) equity value is positively affected by permanent earnings risk.

The remainder of the paper is organized as follows. Section 2 develops the dynamic capital structure model with heterogeneous prior beliefs and derive the equilibrium. Section 3 provides the capital structure implications of the heterogeneity in beliefs and risk factors both through an analytical proof (to be added later) and through a numerical analysis based on the calibration of the model. To obtain further implications of the model, we apply the model to the sample of firms that go public. Section 4 concludes the paper.

## 2 The Model

We consider an all-equity firm whose manager, owning an equity stake  $g_{initial} \in (0, 1)$  in the firm, intends to undertake a positive net present value project with an initial capital investment  $I > 0$ . To obtain financing for the project, the manager approaches the public debt and equity markets, which are assumed to be competitive. In particular, we may regard the equity financing as an initial public offering of the privately held company, thereby leading to changes in its ownership structure. Once the project is implemented, the firm generates a base earnings flow based on its existing assets in subsequent periods. Moreover, the project allows the firm to make an additional earnings flow through the actions of the manager and shareholders, as will be specified in the manager's long-term contract with shareholders.

The total earnings *before interest, taxes and the manager's compensation*(EBITM) are observ-

able and contractible, and are distributed among all the firm’s claimants: the manager, shareholders, debt-holders, and the government (through taxes). We ignore personal taxes for simplicity and assume that the corporate tax rate is a constant  $\tau \in (0, 1)$ . Security issuance costs are negligible, and the risk-free interest rate,  $r$ , is a constant and the same for all market participants.

Our model assumes a finite horizon framework with equally spaced dates,  $0, 1, 2, \dots, T$ . With the possibility of bankruptcy at a time denoted by  $T_b < T$ , the long-term debt contract is due at date  $T$ . If no bankruptcy occurs before date  $T$ , the manager’s contract also continues until then, because the firm does not need additional capital investments as its business matures. However, shareholders continue to operate the firm generating the base earnings thereafter as long as doing so gives them a positive continuation value.

## 2.1 The Firm’s Total EBITM Flow

Following the definition of the *incremental termination payoff* of a project in Giat et al (2009), we assume that the firm’s total EBITM flow in any period  $[i, i + 1]$ ,  $Q_{i+1}$ , for  $i < T_b$ , is the sum of the *base earnings* and the *discretionary earnings*, as presented below. The former is a stochastic component that is unaffected by the actions of the manager and shareholders, whereas the latter is a deterministic component that depends on the physical capital investment by the shareholders,  $k_i$ , and the human capital investment (effort) by the manager,  $\eta_i$ .

$$Q_{i+1} = \overbrace{\Theta + S_{i+1}}^{\text{Base Earnings}} + \overbrace{Ak_i^\alpha \eta_i^\beta}^{\text{Discretionary Earnings}}, \quad i < T_b. \quad (1)$$

The first term of the base earnings,  $\Theta$ , represents the project’s *core output*. As a key assumption in our model, the manager (insiders) and shareholders (outside investors) have imperfect information and may have heterogeneous beliefs about its value. At date zero, their respective priors on  $\Theta$  are normally distributed with different mean values,  $\mu_0^M$  and  $\mu_0^S$ , but with the same mean assessment errors,  $\sigma_0^2$ .<sup>1</sup> As the earnings process is observed by both the manager and shareholders over time, their posterior distributions on  $\Theta$  at each date  $i$  are updated from their initial priors

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<sup>1</sup>Although we mainly discuss the manager’s relative optimism, which means that  $\mu_0^M$  is greater than  $\mu_0^S$ , the model does not exclude the opposite case in which the manager is relatively pessimistic compared to shareholders. In addition, unlike Giat et al (2009), we do not consider the true mean of the core output, not only because the relative difference plays a key role in the equilibrium of the model, but also because it can be more appropriate to derive the firm’s variables based on the beliefs of the capital markets in matching them with actual data.

in a Bayesian manner as  $N(\mu_i^M, \sigma_i^2)$  and  $N(\mu_i^S, \sigma_i^2)$ , respectively, where the detailed equations for  $\mu_i^j, j = M, S$  and for  $\sigma_i^2$  are given in Giat et al (2009).

The difference between their updated mean assessments of  $\Theta$ , denoted by  $\Delta_i$ , is referred to as the *degree of manager optimism* at date  $i$ . Note that there is no difference in the updated variance of  $\Theta$  and that this variance is termed the *transient* risk of earnings (or the *uncertainty* of the core output) because it diminishes as the firm's earnings flow is observed. It is also easily seen that the degree of manager optimism also deterministically declines over time.

The second component of the base earnings,  $S_{i+1}$ , is a random variable that is normally distributed with mean 0 and variance  $s^2$ , serially uncorrelated, and independent of  $\Theta$ . The variance of this variable can be interpreted as the *intrinsic* risk of earnings, in that unlike the transient risk, it remains in the firm's earnings process permanently.

As in Leland (1998), bankruptcy occurs endogenously when the firm in financial distress fails to raise sufficient equity capital to meet its debt obligations. Upon bankruptcy, the debt-holders get control and ownership of the firm, and the existing shareholders receive nothing, following the absolute priority of debt. As in the literature, we consider bankruptcy costs by assuming that after bankruptcy the firm experiences a constant reduction in the future base earnings and that there are no discretionary earnings through the manager's contract as the management is replaced by the debt-holders, which reflects her personal bankruptcy costs. Accordingly, the EBITM flow in the post-bankruptcy period is given by

$$Q_{i+1} = \overbrace{\Theta + S_{i+1}}^{\text{Base Earnings}} - \varsigma. \quad i \geq T_b, \quad (2)$$

In the above,  $\varsigma$  represents a certain loss in the firm's operating earnings, whose size is determined in such a way that the post-bankruptcy value of the firm at date  $T_b$  is a fraction  $(1 - \rho) \in (0, 1)$  of its asset value at that date.

## 2.2 Payoffs to the Firm's Claimants

Assuming that  $T$  is sufficiently long, we can regard all long-term debts issued at date zero as a constant flow of coupon payments to the debt-holders,  $d$ . The optimal coupon payment is endogenously determined by the manager at date zero. It is shareholders, however, who determine at each date whether to continue serving the firm's debt payments depending on their continuation value. In addition to the decision of the time of bankruptcy, the manager's contract with shareholders, as will be detailed in the next section, specifies the manager's compensation payments,  $c^m$ . For such

a contract, the payoffs to the firm's stakeholders are given as follows. In any period  $[i, i + 1]$ , the total after-tax earnings is

$$\begin{aligned} c_i^f &= (1 - \tau)Q_{i+1} + \tau d, & i < T_b, \\ c_i^f &= (1 - \tau)Q_{i+1}, & i \geq T_b. \end{aligned} \quad (3)$$

The above reflects the fact that corporate taxes are incurred on earnings net of debt interest payments. Next, the payoff to debt-holders before bankruptcy is the pre-determined coupon payment, but their payoff in the post-bankruptcy period is the total after-tax earnings because they are the only remaining claimants as the new shareholders of the all-equity firm:

$$\begin{aligned} c_i^d &= d, & i < T_b, \\ c_i^d &= c_i^f, & i \geq T_b. \end{aligned} \quad (4)$$

Consistent with the literature that deals with principal-agent incentive contracting problems, the payoff to the manager before bankruptcy is restricted to have an affine form on the firm's earnings with  $a_i$ , the *performance-invariant* compensation, and  $b_i$ , the *pay-performance sensitivity*. To be exact, as in Bhagat et al (2009), the manager's compensation is expressed in terms of earnings net of interest payments and taxes, which prevents our model from having a direct relation between corporate tax rate and inside equity stake in our implementation of the manager's contract through financial securities. In the post-bankruptcy period, on the other hand, the manager gets zero because the manager, who specializes in the firm, will leave the firm upon bankruptcy and has only alternatives whose payoffs are normalized to zero.

$$\begin{aligned} c_i^m &= a_i + b_i(1 - \tau)(Q_{i+1} - d), & i < T_b, \\ c_i^m &= 0, & i \geq T_b. \end{aligned} \quad (5)$$

Finally, the payoff to shareholders before bankruptcy is the total after-tax earnings net of payments to the manager and debt-holders, whereas they receive nothing after bankruptcy by the priority of debt:

$$\begin{aligned} c_i^s &= c_i^f - c_i^m - c_i^d, & i < T_b, \\ c_i^s &= 0, & i \geq T_b. \end{aligned} \quad (6)$$

### 2.3 The Manager's Contract

In this section, we consider the manager's optimal long-term contract with shareholders for a given long-term debt structure,  $d$ . The contract describes the manager's effort, shareholders' incremental capital investments, payoffs to be received by both parties, and time of termination. We assume that outside investors are risk-neutral, while the manager has constant absolute risk averse (CARA) risk preference represented by a negative exponential utility function,  $u(x) = -\exp(-\lambda x)$ . As explained in Bhagat et al (2009), the contracting environment is incomplete, and only single-period contracts are enforceable. Accordingly, the contract must be *sequentially optimal*, that is, it must be optimal in every continuation contracting problem over the remaining periods, which is why we solve for the optimal long-term contract using a dynamic programming approach.

The manager has the bargaining power over shareholders because they supply capital competitively in the public stock market. Accordingly, the contract is optimally chosen by the manager, maximizing her expected utility subject to incentive compatibility constraints with respect to her effort and dynamic participation constraints for shareholders. The manager's incentive compatible constraints ensure that the manager will follow the effort choice specified by the contract at each date because it maximizes her expected utility when other contract variables are given. The dynamic participation constraints for shareholders guarantees the *reservation payout flow* to them that is what they could receive from the firm's assets without the contract with the manager,  $(1 - \tau)(\Theta + S_{i+1} - d)$  in any period  $[i, i + 1]$ . For the shareholders to accept the contract, their continuation value under the contract at each date must be at least as great as the value that would be derived if they received the reservation payout flow in each subsequent period before bankruptcy.

Let  $\{\mathcal{F}_i\}$  denote the information generated by the history of the firm's total EBITM process and discretionary earnings flow until date  $i$ . Given a long-term debt structure  $d$  of the firm, a feasible contract satisfying the above both constraints is described by the set of  $(a_i, b_i, k_i, \eta_i, T_b)$ , where  $a_i, b_i, k_i, \eta_i$  are  $\{\mathcal{F}_i\}$ -adapted and the optimal termination time of the contract  $T_b$ , that is, the time of the firm's bankruptcy, is an  $\{\mathcal{F}_i\}$ -stopping time. We now solve for the optimal contractual variables over time first by dealing with the problem for the contractually final period  $[T - 1, T]$  and then by working out problems for earlier dates recursively.

### 2.3.1 The Contracting Problem for Period $[T - 1, T]$

Suppose that the firm is solvent as of date  $i = T - 1$ . The manager's conditional expected utility from the payoff over the period including her disutility of effort is given by

$$CEU_i := E_i^M \left[ -\exp \left( -\lambda(c_i^m - \kappa\eta_i^\gamma) \right) \right] = -\exp \left( -\lambda\Pi(a_i, b_i, k_i, \eta_i) \right), \quad (7)$$

where

$$\Pi(a_i, b_i, k_i, \eta_i) = a_i + (1 - \tau) \left( \mu_i^M + Ak_i^\alpha \eta_i^\beta - d \right) b_i - \kappa\eta_i^\gamma - 0.5\lambda(1 - \tau)^2(\sigma_i^2 + s^2)b_i^2. \quad (8)$$

The above equation, (7), is derived using (1), (5), and the fact that  $\Theta + S_{i+1} \sim N(\mu_i^M, \sigma_i^2 + s^2)$  under the manager's belief. Also, note that maximizing the manager's conditional expected utility is equivalent to maximizing  $\Pi(a_i, b_i, k_i, \eta_i)$ .

A feasible contract, as noted above, must be incentive compatible for the manager to exert the specified effort in the contract that maximizes her conditional expected utility (7), when the other contractual variables  $(a_i, b_i, k_i)$  are given. The manager's optimal effort,  $\eta_i$ , is derived as <sup>2</sup>

$$\eta_i = \eta(b_i, k_i) = \left( \frac{A(1 - \tau)\beta k_i^\alpha b_i}{\gamma\kappa} \right)^{\frac{1}{\gamma - \beta}}. \quad (9)$$

Moreover, the participation constraint for shareholders must be satisfied by the remaining contractual variables  $(a_i, b_i, k_i)$ , which is given by

$$CV_i := E_i^S [c_i^s - k_i] \geq E_i^S [(1 - \tau)(\Theta + S_{i+1} - d)]. \quad (10)$$

More exactly, the base earnings flow beyond date  $T$  that the mature firm will generate must be included in the equation for the continuation value of shareholders at date  $i$ ,  $CV_i$ , which is defined as their expected payoff with respect to their beliefs if they are determined not to terminate the contract at this date. However, since this assumption on the activity of the mature firm after date  $T$  is mainly necessary for the final condition to work backwards over the periods in our numerical analysis, we do not include the earnings flow after date  $T$  in the theoretical analysis. It is obvious that the above participation constraint must be satisfied with equality to maximize the manager's objective function, which derives the contractual parameter  $a_i$  as a function of the other two variables:

$$a_i = a(b_i, k_i) = -(1 - \tau)(\mu_i^S - d)b_i + (1 - \tau)Ak_i^\alpha \eta(b_i, k_i)^\beta (1 - b_i) - k_i. \quad (11)$$

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<sup>2</sup>For the same reason as in Giat et al (2009), we assume the condition on the parameters,  $(1 - \alpha)\gamma/\beta > 2$ , which also guarantees a unique optimal effort choice.

By substituting the manager's effort function (9) and the manager's performance-invariant compensation function (11) into  $\Pi(a_i, b_i, k_i, \eta_i)$ , the objective function to be maximized simplifies to

$$\Pi(b_i, k_i) = (1 - \tau)\Delta_i b_i + \phi(b_i)k_i^{\frac{\alpha\gamma}{\gamma-\beta}} - k_i - 0.5\lambda(1 - \tau)^2(\sigma_i^2 + s^2)b_i^2. \quad (12)$$

In (12),

$$\phi(b_i) := (A(1 - \tau))^{\frac{\gamma}{\gamma-\beta}} \left(\frac{1}{\kappa}\right)^{\frac{\beta}{\gamma-\beta}} \left(\frac{\beta b_i}{\gamma}\right)^{\frac{\beta}{\gamma-\beta}} \left(1 - \frac{\beta b_i}{\gamma}\right). \quad (13)$$

It now remains to determine the optimal capital investment of shareholders and the manager's optimal pay-performance sensitivity that maximize the manager's objective function, (12). Given a fixed value of  $b_i \in (0, \gamma/\beta)$ , which guarantees a positive solution for the capital investment, we obtain the optimal investment as a function of the manager's pay-performance sensitivity by setting the partial derivative of (12) with respect to  $k_i$  equal to zero:

$$k(b_i) = \left[ \frac{\alpha\gamma}{\gamma - \beta} \phi(b_i) \right]^{\frac{\gamma-\beta}{(1-\alpha)\gamma-\beta}}. \quad (14)$$

By this optimal investment function(14), the manager's problem reduces to the optimal choice of the pay-performance sensitivity:

$$b_i^* = \arg \max_{b_i \geq 0} F_i(b_i), \quad (15)$$

where  $F_i(b_i) = (1 - \tau)\Delta_i b_i + Bk(b_i) - 0.5\lambda(1 - \tau)^2(\sigma_i^2 + s^2)b_i^2$  with  $B = \frac{(1-\alpha)\gamma-\beta}{\alpha\gamma}$ .

Consequently, the manager's optimal contracting problem for period  $[T - 1, T]$ , conditional on the firm's solvency at date  $T - 1$ , is completely specified by  $b_i^*$ . The optimal capital investment of shareholders  $k_i^*$  is  $k(b_i^*)$ , manager's optimal effort  $\eta_i^*$  is  $\eta(b_i^*, k_i^*)$ , and manager's performance-invariant compensation  $a_i^*$  is  $a(b_i^*, k_i^*)$ . As we mentioned, the results only hold when the firm is solvent in the beginning of this period. If shareholders expect a negative or zero continuation value after observing the earnings flow until the previous period, the contract is optimally terminated at this date, that is,  $T_b = T - 1$ .

### 2.3.2 The Inductive Step

We now set  $i = T - 2$  and consider the optimal contracting problem at date  $i$ , provided that the contract has not been terminated until this date. The manager's conditional expected utility is

then written by the law of iterated expectations as below:

$$\begin{aligned} CEU_i &= E_i^M \left[ - \exp \left( - \lambda \left( \sum_{j=i}^{T_b-1} e^{-r(j-i)} (c_j^m - k\eta_j^\gamma) \right) \right) \right] \\ &= - \exp \left( - \lambda \Pi (a_i, b_i, k_i, \eta_i) \right) (-\widehat{CEU}_{i+1}^*). \end{aligned} \quad (16)$$

The last term  $-\widehat{CEU}_{i+1}^*$  is positive and unaffected by the contractual variables to be optimally chosen for the current period because the term is one, if bankruptcy occurs at date  $i + 1$ , and if no bankruptcy, it is the modified conditional expected utility at that date (7) due to the discount factor that is already determined in the process of backward induction.<sup>3</sup> Moreover, the participation constraint for shareholders is given by

$$CV_i := E_i^S [c_i^s - k_i + e^{-r} \max\{CV_{i+1}, 0\}] \geq E_i^S \left[ \sum_{j=i}^{T_b-1} e^{-r(j-i)} (1 - \tau) (\Theta + S_{j+1} - d) \right]. \quad (17)$$

Because (10) for  $CV_{i+1}$  must be satisfied with equality, if  $CV_{i+1} > 0$ , and  $T_b = i + 1$ , if  $CV_{i+1} \leq 0$ , we can see that (17) is identical to (10). Furthermore, whether the firm will go bankrupt at date  $i + 1$  is also not related to the deterministic decision for the current period, but to the random realizations of the base earnings. As a consequence, the optimal contractual parameters for period  $[T - 2, T - 1]$  can be determined in the same manner as in the contracting problem for period  $[T - 1, T]$ , and the same argument applies to any period  $[i, i + 1]$  for  $i < T - 2$ . Likewise, before applying the results of the optimal contract for period  $[i, i + 1]$ , we must make sure that the contract has not been terminated in the beginning of this period, that is, the firm is still solvent until then.

### 2.3.3 The Optimal Contract and Its Implications

The Theorem 1 summarizes the manager's optimal contract with shareholders we have derived in the prior sections.

#### Theorem 1 (The Manager's Optimal Contract)

*Conditional on the firm being solvent on its long-term debt structure  $d$  at date  $i \leq T - 1$ , we have the following:*

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<sup>3</sup>For  $i = T - 2$  and  $i + 1 \neq T_b$ ,

$$\widehat{CEU}_{i+1}^* = - \exp \left\{ - \lambda e^{-r} \left( (1 - \tau) \Delta_{i+1} b_{i+1}^* + Bk(b_{i+1}^*) - 0.5 \lambda e^{-r} (1 - \tau)^2 (\sigma_{i+1}^2 + s^2) b_{i+1}^{*2} \right) \right\}.$$

- (a) *the manager's pay-performance sensitivity  $b_i^*$  solves  $b_i^* = \arg \max_{b \geq 0} F_i(b)$ , where  $F_i(b) = (1 - \tau)\Delta_i b + Bk(b) - 0.5\lambda(1 - \tau)^2(\sigma_i^2 + s^2)b^2$ .*
- (b) *the incremental capital investment of shareholders is  $k_i^* = k(b_i^*)$ .*
- (c) *the manager's effort is  $\eta_i^* = \eta(b_i^*, k_i^*)$ .*
- (d) *the manager's performance-invariant compensation is  $a_i^* = a(b_i^*, k_i^*)$ .*
- (e) *the termination time of the contract, i.e., the firm's time of bankruptcy, solves the following optimal stopping problem:*

$$T_b = \arg \max_{\tau \leq T} E_0^S \left[ \sum_{j=0}^{\tau-1} e^{-r(j-i)} (1 - \tau) (\Theta + S_{j+1} - d) \right], \quad (18)$$

where the maximization is over all  $\{F_i\}$ -stopping times  $\tau \leq T$ .

Given the manager's and shareholders' beliefs about the core output at the beginning of period  $[i, i + 1]$ ,  $N(\mu_i^M, \sigma_i^2)$  and  $N(\mu_i^S, \sigma_i^2)$ , respectively, the optimal contractual parameters that are conditional upon continuation in this period,  $(a_i^*, b_i^*, k_i^*, \eta_i^*)$ , are deterministically chosen by following the procedure stated in Theorem 1, where the optimal investment function of shareholders and the manager's effort and performance-invariant compensation functions are defined in Section 2.3.1. Since the manager's optimal contract in the above theorem correspond to the dynamic equilibrium in Giat et al (2009), we can apply the results of sensitivity analysis of the equilibrium dynamics that are well presented in their paper to our results with some minor changes concerning the corporate tax rate they do not consider.

The manager's pay-performance sensitivity, which reflects the risk-averse manager's incentives to share the risk of the firm's random earnings process with shareholders, increases with the initial degree of heterogeneity in beliefs,  $\Delta_0$ , and declines with the two risk factors of the firm's earnings - the initial transient risk  $\sigma_0$  and intrinsic risk  $s$ . The former observation comes from the fact that the more optimistic the manager is the more she overvalues the firm's future earnings, thereby putting more weight on the performance-variant component in her compensation. The latter observation, on the other hand, can be explained by the increase in the manager's costs of risk-sharing with shareholders.

The effects of these parameters on the optimal capital investment of shareholders and the manager's effort depend on the relative magnitudes of the degree of heterogeneity in beliefs to the price of risk. If the degree of heterogeneity is less than the price of risk, the actions both by

the manager and by shareholders increase (decrease) with the heterogeneity in beliefs (with risk parameters). If the manager is too optimistic compared to shareholders, however, the incremental capital supply of shareholders decreases (increases) with the degree of heterogeneity (with risk parameters), whereas the manager's effort varies non-monotonically. In order to understand the effects of the heterogeneity in beliefs and that of risk factors on capital structure of the firm, the properties of the optimal contractual variables we have discussed here must be considered.

## 2.4 Dynamic Capital Structure

### 2.4.1 The Manager's Financing Choice

We now characterize the manager's optimal choice of long-term debt, which was given in the manager's optimal contracting problem in the previous section, but must be endogenously determined with the manager's contract for the equilibrium of our model. As mentioned earlier, the optimal long-term debt contract specifies the optimal perpetual coupon payment to the debt-holders,  $d^*$ . The manager needs to consider both the initial payoff at date zero, her share of the initial all-equity firm's surplus generated from external financing, and the continuation payoff from her compensation payments through the contract with shareholders over the subsequent contracting periods, as detailed in Bhagat et al (2009).

The firm's surplus generated from external financing for a new project can be interpreted as the increase in the market value of the firm at date zero right after the announcement of the plan, which may be measured by the concept of net present value of the project, that is, the difference between the present value of the firm's earnings flow to be generated by the project and the initially required investment outlay  $I$ . The firm's earnings here is the after-tax base earnings to consider the benefit of external financing itself.

$$SUR_0 = E_0^S \left[ \sum_{i=0}^{T_b-1} e^{-ri} \{ (1-\tau)(\Theta + S_{i+1}) + \tau d \} - I \right]. \quad (19)$$

Note that we measure the surplus as the market value based on the beliefs of shareholders (outside investors) about the core output of the project.

In addition to the initial payoff at date zero,  $g_{initial}[SUR_0]$ , the manager expects the continuation payoff obtained from the contract with shareholders, which is the sum of her future compensation payments net of personal costs of exerting the optimal effort:

$$M_0 = \sum_{i=0}^{T_b-1} e^{-ri} (c_i^{m*} - \kappa \eta_i^{*\gamma}). \quad (20)$$

Therefore, the optimal long-term debt coupon payment,  $d^*$ , solves the following optimization problem:

$$\begin{aligned}
d^* &= \arg \max_{d \geq 0} E_0^M \left[ - \exp \left( - \lambda \left\{ \overbrace{g_{initial} [SUR_0]}^{\text{Initial Payoff}} + \overbrace{M_0}^{\text{Continuation Payoff}} \right\} \right) \right] \\
&= \arg \max_{d \geq 0} \exp \left( - \lambda \{g_{initial} [SUR_0]\} \right) CEU_0^*.
\end{aligned} \tag{21}$$

The second line above is written by borrowing the definition of the conditional expected utility from (16). In the above problem, the manager chooses the firm's optimal coupon at date zero in a way of maximizing the manager's expected utility that is derived from her two types of payoffs. We should notice the well-known trade-off between the benefit of debt tax shields and the potential bankruptcy costs that will be incurred upon default on debt obligations. By choosing greater long-term debt, the manager can increase the surplus from external financing because interest payments to debt are tax-deductible, thereby increasing her initial payoff. In this case, the manager also needs to consider a detrimental effect of bankruptcy costs both on her initial payoff and on her continuation payoff, as the firm is more likely to default. In sum, as we have seen from the previous and this sections, the equilibrium of our model is characterized first by deriving the manager's optimal contract for a given long-term debt structure and then by solving for the manager's optimal choice of long-term debt.

#### 2.4.2 Implementation of the Manager's Contract

We have considered only equity and long-term debt for the firm's capital structure, but we now assume that it consists of inside equity, outside equity, long-term debt issued at date zero, and dynamic short-term lending or borrowing. Then we can implement the manager's compensation structure specified in the optimal contract with shareholders through dynamic holdings in financial securities, specifically inside equity and short-term lending or borrowing. By (3) and (4), we can rewrite the manager's payoff in period  $[i, i + 1]$  before bankruptcy (5) as

$$c_i^m = b_i^* [c_i^f - c_i^d + \bar{a}_i], \tag{22}$$

where

$$\bar{a}_i = a_i^* / b_i^*.$$

The above (22) implies the manager's optimal compensation structure can be implemented through an inside equity stake,  $b_i^*$ , and an additional cash flow,  $\bar{a}_i$ , to all equity holders (inside and outside)

in each period. Depending on their sign, the cash flows  $\bar{a}_i$  could be viewed as dynamic short-term lending or borrowing (or dynamic payments into or out of cash reserve). The long-term debt issued at date zero determines the firm's base leverage, while the short-term debt reflects dynamic and stochastic fluctuations about the base leverage level. The market values of long-term debt, outside equity, and short-term debt at any date  $i < T_b$  are defined as follows:

$$\begin{aligned}
\text{Long-Term Debt Value } (LDV_i) &= E_i^S \left[ \sum_{j=i}^{T-1} e^{-r(j-i)} c_j^d \right] \\
\text{Outside Equity Value } (OEV_i) &= E_i^S \left[ \sum_{j=i}^{T_b-1} e^{-r(j-i)} (1 - b_j^*) (c_j^f - c_j^d + \bar{a}_j) \right], \\
\text{Short-Term Debt Value } (SDV_i) &= E_i^S \left[ \sum_{j=i}^{T_b-1} e^{-r(j-i)} (-\bar{a}_j) \right].
\end{aligned} \tag{23}$$

At any date  $i \geq T_b$ , on the other hand, only the outside equity value is defined in the same manner as the long-term debt value above because the firm returns to an all-equity firm with the existing debt-holders as the new shareholders.

### 3 The Effects of Heterogenous Beliefs and Risks

In this section, we investigate how heterogeneous beliefs between the manager and outside investors and the transient and intrinsic risks of the firm's earnings affect the firm's capital structure. Although the effects on the firm's long-term debt can be analytically investigated, those on the firm's short-term debt are ambiguous for general parameter values. Accordingly, we also numerically analyze the effects after calibrating the model to a large sample of firm-year observations from Compustat.

#### 3.1 Analytical Proof

To be added soon.

#### 3.2 Numerical Analysis

In this analysis, we first estimate the basic structural parameters of the model using the general firm data from Compustat. Then we perform a sensitivity analysis from the baseline parameters to investigate how a firm's capital structure varies with the model parameters of interest. Finally,

we apply the model to the data of firms going public that are more consistent with the main idea of the model because the firms are believed to have greater differences in beliefs.

### 3.2.1 Model Calibration

**Basic Economic Parameter Values:** We set the risk-free rate  $r$  to 4.65%, which is equal to the median rate of 3 month Treasury bills over our sample period of 1993 – 2003, as reported in FRED (Federal Reserve Economic Data), and the effective corporate tax rate  $\tau$  to 0.15 from the estimates of Graham (2000). The maximum contracting date as well as the maturity date of long-term debt issued at date zero,  $T$ , is set to 30 years, assuming the period length of one year. In the discretionary output following a Cobb-Douglas production function,  $\alpha$ , is set to 0.3. If we assume that  $\beta$ , which is jointly estimated with  $\gamma$  in the calibration, is  $1 - \alpha$ ,  $\alpha$  is the capital share of output and 0.3 is a familiar parameter value from the U.S. macroeconomic data. Finally, we assign 0.15 to the bankruptcy cost parameter,  $\rho$ , that is the midpoint of the estimates reported in Andrade and Kaplan (1998).

**Definitions of the Statistics:** We first need to define some variables for our calibration. As for the long-term debt and short-term debt values, we follow the definitions stated in the previous section, except that we need to include the earnings flow beyond date  $T$  for the final condition to dynamic equations. Next, we proxy the *asset value* as the firm value, if it were hypothetically un-levered and raised the initial project outlay through equity, and if there were no additional actions through the manager’s contract. It thus follows that the asset value at any date  $i$ ,  $AV_i$ , is the present value of the stream of the base earnings after tax payments.

$$AV_i = E_i^S \left[ \sum_{j=i}^{\infty} e^{-r(j-i)} (1 - \tau)(\Theta + S_{j+1}) \right]. \quad (24)$$

All of our statistics that will be used in the calibration except for the inside equity stake are scaled by the asset value defined above. Similarly, the total value of the firm is measured as the present value of the stream of the total after-tax earnings,  $c_i^f$ , which is specified in (3),

$$FV_i = E_i^S \left[ \sum_{j=i}^{\infty} e^{-r(j-i)} c_i^f \right]. \quad (25)$$

Note that the flow of  $c_i^f$  has a discontinuity at bankruptcy, which implies that the firm value incorporates the possibility of bankruptcy. More specifically, we can see that the total value of the

firm is composed of the asset value and the probability-adjusted manager's stake, debt tax-shields, and bankruptcy costs.

Using these quantities, especially at date zero, we select the statistics to be matched with the actual data: (i) the ratio of long-term debt to asset value,  $LDV_0/AV_0$ ; (ii) the ratio of short-term debt to asset value,  $SDV_0/AV_0$ ; (iii) the ratio of total firm value to asset value,  $FV_0/AV_0$ ; (iv) the ratio of incremental investment to asset value,  $k_0/AV_0$  (v) the ratio of EBITM to asset value,  $E_0^S[Q_1]/AV_0$ ; (vi) the ratio of net income to asset value:  $E_0^S[c_0^f - c_0^d - c_0^m]/AV_0 = E_0^S[c_0^s]/AV_0$ , and, finally, (vii) inside equity stake,  $b_0^*$ .

**Data Description:** In the calibration, we determine the key parameter values of the model by matching the model-predicted statistics defined above with the median values of the statistics in the data. To obtain the median values of the observed statistics, we use an unbalanced panel data set over the period of 1993-2003, which are reported in Standard and Poor's Compustat industrial files. We first delete observations with missing variables or with total assets (Compustat item #6) or investment (Compustat item #30) of either zero or a negative number. Because of the generality of the model, we omit all regulated or financial firms, whose primary SIC classification is between 4,900 and 4,999 or between 6,000 and 6,999. Accordingly, we end up with a sample of firm-year observations with the total number of 59,589 and simply regard them as a pooled data set, as if each observation represented one firm. Although the entire sample is used in the first analysis including the sensitivity analysis, we select a sub-sample of 3,440 observations for the analysis of the firms going public that are traced immediately after the offering.

In the sample for each analysis, we extract variables to compute the statistics. Specifically, the long-term debt value is obtained from Compustat item #9, while the net short-term debt value is computed as debt in current liabilities (Compustat item #34) minus debt due in one year (Compustat item #44) minus cash (Compustat item #1), as in Bhagat et al (2009). The total value of the firm is measured by the sum of equity value, which is the closing stock price (Compustat item #199) multiplied by the number of common shares outstanding (Compustat item #25), and the total value of debt, which is long-term debt (Compustat item #9) plus short-term debt (Compustat item #34) minus cash (Compustat item #1). The EBITM in the model corresponds to operating income (Compustat item #13), and we use net income (Compustat item #172) to proxy the return on assets.

The computed variables including investment (Compustat item #30) are all divided by the asset value (Compustat item #6), and their median values of the ratios are used to be matched with

the model-predicted statistics. Lastly, the median value of managerial equity ownership, which is not only used for one of the model parameters, the manager’s initial equity stake in the all-equity firm at date zero,  $g_{initial}$ , but is also the last statistic to be matched with the model prediction. Since the ownership information corresponding to our sample is not available, we use the estimates reported in other studies. In the first analysis for the full sample, we use the median percentage of ownership of insiders (officers and directors), 14.4%, that Holderness, Kroszner, and Sheehan (1999) estimate for exchange-listed firms in 1995 from Compact Disclosure. In the second analysis of initial public offering (IPO) firms, we borrow the ownership estimates of the chief executive officer (CEO) that Mikkelsen, Partch, and Shah (1997) report for a sample of 283 initial public offerings by U.S. companies in the years 1980-1983. To reflect the fact that an initial public offering leads to significant changes in the ownership of a company’s stock, as observed in their analysis, we set  $g_{initial}$  to 24.8%, which is the median stake of the CEO prior to the offering, and the observed statistic for the inside equity stake in the model to 15.9%, which is the median value of the CEO stake that is followed immediately after the offering.

***Implementation of the Calibration:*** In our model, the set of the parameters to be estimated is

$$\pi = (A, s, \mu_0^S, \Delta_0, \sigma_0, \lambda, \kappa, \gamma/\beta).$$

As discussed above, there are only seven model-predicted statistics to be matched with the observed ones in the data. Since at least the same number of statistics as the number of unknown parameter values are required for identification, we directly estimate the intrinsic risk of the firm’s earnings,  $s$ , from the data. Provided that the transient risk is the assessment error by the manager or by shareholders and disappears over time, the intrinsic risk can be thought of as the main risk in the firm’s total EBITM, and, therefore, we proxy this risk by the median value of three-year standard deviations of operating income per share in the data. To have this variable for the period of 1993-2003, we need to additionally use the data of 1992 and 2004. Also, note that we do not estimate the parameters,  $\beta$  and  $\gamma$ , because all observable variables only depend on their ratio.

We now look at how to implement the calibration to determine the seven parameters of the model. We find the parameter values minimizing the difference between the predicted and observed values of the defined statistics. In doing so, we use the Nelder-Meade optimization method available in MATLAB, which is a simplex search method, and thus appropriate for our optimization that cannot use numerical or analytic gradients of the objective function and that has discontinuity in values because of the likelihood of bankruptcy. Because this method, like other usual numerical

methods, may often give a local minimum solution, we need to do the optimization with several different initial guesses for the parameter values and take the best of the resulting local optima as the global solution. In particular, by randomizing the initial guess, we may overcome some difficulty in choosing the global optimum in the wide ranges of the parameter values.

We describe how to compute the model-predicted statistics, which are defined as date zero values, for a candidate parameter vector  $\pi$  chosen by random sampling. First, we deterministically solve for the manager's optimal contract variables at each date  $i$  for  $i = 0, 1, 2, \dots, T - 1$ , which determine the firm's discretionary earnings and will be applied only if the firm is not bankrupt before date  $i$ .

Then we implement through a discretization approach the base earnings that are the stochastic part in the firm's earnings. Using a discrete lattice, we set up the stochastic evolution of  $\mu_i^S$  for  $i = 1, 2, \dots, T$ , which is updated in a Bayesian manner from  $\mu_0^S$ . Giat et al (2009) use this implementation in their simulation in order to determine the optimal stopping time by the principal, which is one of the reasons why we model the process of the mean assessment of the core output by shareholders,  $\mu_i^S$ , instead of the process of  $\mu_i^M$ . Another reason for considering the process of  $\mu_i^S$  is that most of the statistics we will fit with the actual data are based on the market value, such as debt values and firm value, which must be computed under the beliefs of shareholders in our model.

At date zero, shareholders assess the core output and estimate its mean as  $\mu_0^S$ . Let  $n(i)$  denote the number of states at date  $i > 0$  and  $\mu_{i,j}^S$  their updated assessment at the  $j^{\text{th}}$  state at date  $i$ ,  $j = 1, \dots, n(i)$ , where different states for the posterior beliefs of shareholders result from different random realizations of the base earnings over period  $[i - 1, i]$ . The lattice is designed so that the minimum value at date  $i$ ,  $\mu_{i,1}^S$ , is  $\kappa (= 2.5)$  standard deviation below the minimum value at date  $i - 1$ , while the maximum value at date  $i$ ,  $\mu_{i,n(i)}^S$ , is  $\kappa$  standard deviation above the maximum value at date  $i - 1$  (Note that if  $i = 1$ , the minimum and maximum values are  $\kappa = 2.5$  standard deviation below and above  $\mu_0^S$ ):

$$\mu_{i,1}^S = \mu_{i-1,1}^S - \kappa\sigma_{i-1}^\mu \quad \text{and} \quad \mu_{i,n(i)}^S = \mu_{i-1,n(i-1)}^S + \kappa\sigma_{i-1}^\mu,$$

where  $\sigma_{i-1}^\mu$  is the standard deviation of the evolution of  $\mu_i^S$  and is derived by the Bayesian equations for  $\mu_i^S$  and for  $\sigma_i^2$  as follows:

$$(\sigma_i^\mu)^2 = \text{Var}_i[\mu_{i+1}^S - \mu_i^S] = \frac{s^2}{[(s/\sigma_0)^2 + i + 1][(s/\sigma_0)^2 + i]}. \quad (26)$$

After the minimum and maximum values are determined, the values for the remaining  $n(i) - 2$  states are equally spaced between them. The number of states in the lattice increases linearly from period to period, that is,  $n(i) = Mi$  for  $i > 0$ , where we set  $M = 20$ . Even though the manager's contract and the long-term debt obligation are over at most date  $T$ , we get the state values at date  $T$  to obtain final conditions to backward equations aimed at computing values at date zero.

After constructing the lattice of  $\mu_{i,j}^S$  from date 1 to date  $T$ , as described above, we derive the transition probability structure defined in Giat et al (2009) for dynamic programming approach. Let  $p_{i,j}^{i+1,k}$  denote the probability that the mean assessment of the core output will transition from state  $\mu_{i,j}^S$  at date  $i$  to state  $\mu_{i+1,k}^S$  at date  $i + 1$  for all  $i, j$ , and  $k$ . Based on the normality of the assessment, if  $\mu_{i+1,k}^S$  is within  $\pm \kappa \sigma_i^\mu$  from  $\mu_{i,j}^S$ , we set

$$p_{i,j}^{i+1,k} := \Phi \left[ \left( \frac{1}{2} (\mu_{i+1,k}^S + \mu_{i+1,k+1}^S) - \mu_{i,j}^S \right) \frac{1}{\sigma_i^\mu} \right] - \Phi \left[ \left( \frac{1}{2} (\mu_{i+1,k}^S + \mu_{i+1,k-1}^S) - \mu_{i,j}^S \right) \frac{1}{\sigma_i^\mu} \right], \quad (27)$$

where  $\Phi(\cdot)$  denotes the cumulative density function of the standard normal distribution. Otherwise, the transition probability is zero.

Employing both the optimal contractual choices and the settings for the stochastic base earnings process, we now numerically solve the problem for the optimal long-term debt coupon payment, which is determined by the manager's expected utility maximization at date zero, (21). Comparing the manager's expected utilities at date zero for different values of  $d$  through the Nelder-Meade optimization method, we choose the optimal long-term debt structure,  $d^*$ .

For a given  $d$ , to begin with, we need to identify the optimal time of bankruptcy, which is the time when the continuation value of shareholders is equal to or less than zero. As detailed in Giat et al (2009), we obtain the continuation values of shareholders at date  $i$  and state  $j$ ,  $CV_{i,j}$ , based on the state values,  $\mu_{i,j}^S$ . The final condition at date  $i = T$  to work backwards is

$$\begin{aligned} CV_{i,j} &= (1 - \tau) \mu_{i,j}^S / (1 - e^{-r}) \quad \text{if } \mu_{i,j}^S \geq 0, \\ CV_{i,j} &= 0, \quad \text{otherwise.} \end{aligned} \quad (28)$$

It follows the assumption that after the last contracting period, the existing or new shareholders, the only stake-holders of the firm, estimate the firm's future value based on their beliefs at that time, and only if it is positive, they will operate the mature firm generating the base earnings with no growth opportunities, thereby deriving the continuation value as defined above. At earlier dates, however, the continuation values for any state are given by a backward equation

$$CV_{i,j} = (1 - \tau) (\mu_{i,j}^S - d) + e^{-r} \sum_{k=1}^{n(i+1)} p_{i,j}^{i+1,k} \max(CV_{i+1,k}, 0), \quad (29)$$

where  $p_{i,j}^{i+1,k}$  are specified in (27). Since the continuation value function is continuous and increasing in  $mu_{i,j}^S$ , we determine the optimal bankruptcy trigger  $\mu_i^*$  that solves  $CV_i(\mu_i^*) = 0$  by linear interpolation. Thus, if  $\mu_{i,j}^S \leq \mu_i^*$  at each date  $i$ , the firm goes into bankruptcy.

With when (at which states) bankruptcy occurs in mind, we now use dynamic programming to compute the manager's expected utility at date zero that is derived from her initial and continuation payoff. For the manager's initial payoff, the surplus from external financing, (19) must be computed. First, the initial investment outlay,  $I$ , is set equal to the asset value at date zero, (24) for  $i=0$ , and derived by starting from the final condition, in which the asset value at date  $T$  is the same as the continuation value of shareholders,  $CV_T$ , and working backwards through time, which is given for earlier dates by

$$AV_{i,j} = (1 - \tau)\mu_{i,j}^S + e^{-r} \sum_{k=1}^{n(i+1)} p_{i,j}^{i+1,k} AV_{i+1,k}. \quad (30)$$

Second, the date zero value of the cash inflow derived from the project in the surplus equation is similarly computed by backward induction, but while taking the likelihood of bankruptcy into account. Hence, if the bankruptcy occurs, i.e.,  $\mu_{i,j}^S < \mu_i^*$  at date  $i$  and state  $j$ , the value is assigned to  $(1 - \rho)AV_{i,j}$ , reflecting the bankruptcy costs, as we assumed that the post-bankruptcy value of the firm at date  $T_b$  is a fraction  $(1 - \rho) \in (0, 1)$  of its asset value at that date. The manager's initial payoff is  $g_{initial}$  share of this value net of the initial investment outlay.

In addition to the initial payoff, the manager needs to consider her future payout flow, which is reflected in the term  $CEU_0^*$  in (21). Although the manager's contractual variables are deterministically determined over time, due to the likelihood of bankruptcy, we need to similarly compute  $CEU_{i,j}$  at all states and dates by backward induction. As the final condition, this value is one at date  $T$ . For earlier dates, the pre-computed  $\hat{\Pi}(a_i^*, b_i^*, c_i^*, \eta_i^*)$  function value, which is evaluated at the optimal contractual variables and modified with the discount rate, is added in the manager's negative exponential utility function, if no bankruptcy at the state, or nothing is added, if bankruptcy occurs. Considering both the initial and continuation payoffs, the manager optimally choose the firm's long-term debt coupon,  $d^*$ , at date zero. After the optimal long-term debt structure is determined, the model-predicted statistics consisting of date zero values are similarly computed through dynamic programming.

### 3.2.2 Sensitivity Analysis

This section reports the baseline parameter values obtained by the model calibration for general firms and performs a sensitivity analysis based on the estimated baseline values to examine the effects of the parameters of interest on the firm's capital structure.

The results from the calibration for general firms are presented in Tables 1 and 2. Table 1 compares the observed values with the predicted values of the statistics. The model reasonably matches all of the statistics except investment and net income ratios, which are relatively difficult to be matched well because one may need to consider many adjustment factors in measuring these variables for data. The baseline values of the parameters are reported in Table 2, where the calibrated parameters of the model are grouped into "technology", "preference" and "belief" categories. We indirectly infer from the results that managers are relatively optimistic compared to outside investors even though we also considered the possibility of managerial pessimism in the model. Also, we observe the higher initial transient risk than the intrinsic risk directly estimated from the data, which implies that a substantial portion of earnings risk comes from the imperfect information and is transitory. We delay further discussion on other implications from the estimated baseline parameter values for general firms to the next section, which will provide those for firms going public and therefore allow us to compare both the results.

Through the sensitivity analysis in which defined output variables are observed as only one particular parameter value varies while all the other parameters are set to their baseline values, we examine the capital structure implications of the degree of managerial optimism  $\Delta_0$  and the initial transient risk  $\sigma_0$  and the intrinsic risk  $s$  of the firm's earnings. In addition to the long-term debt and short-term debt ratios, we look at variations of the equity value ratio, which is simply computed as the firm value ratio subtracted by the total debt value ratio.

Figure 2 displays the variations of the firm's capital structure variables with  $\Delta_0$ . Even though the long-term debt ratio shows just one drop around its baseline value, it partially argues that long-term debt decreases with managerial optimism. By contrast, the short-term debt ratio increases with this parameter, and the opposite effects on long-term debt and on short-term debt may support the results of Landier and Thesmar (2008).

To understand these observations, we look at the variations of equilibrium contract variables with  $\Delta_0$  in Figure 1. As the degree of managerial optimism increases, the manager overvalues the firm's future earnings relative to shareholders, and the risk-averse manager's incentive of risk

sharing with shareholders, represented by the manager's pay-performance sensitivity, increases. Due to this increased incentive, the manager's effort and shareholders' incremental capital investments also increase with managerial optimism. Since the discretionary earnings from their co-investments go to the manager instead of competitive investors, her continuation value relative to her initial payoff from leveraging the firm at the margin increases, and, therefore, she will choose lower long-term debt to reduce the probability of bankruptcy.

In order to explain the effects on short-term debt, we should consider the two underlying changes in the manager's pay-performance sensitivity and in the long-term debt coupon. As mentioned, the manager's pay-performance sensitivity increases with managerial optimism. By (11), the manager's performance-invariant compensation, other things being equal, increases with the long-term debt coupon (in order to compensate for the decline in the earnings net of interest payments and taxes). Since both the changes from an increase in managerial optimism decrease the ratio  $a_i^*/b_i^*$  in each period, that is, the manager's compensation relies more on the risky component than cash, the firm's cash reserve decreases, and, equivalently, its short-term debt increases with managerial optimism.

One concern about our results is how to interpret the observation that compared to the variations of the short-term debt ratio, the long-term debt ratio varies less sensitively. It could be true that managerial optimism is more related to the firm's cash reserve policies (or credit line balances). Or the difference in variations of these two debt variables could be because of our model assumption that the long-term debt structure is determined only once at date zero. If that is the case, we cannot say the relative impacts on the debt ratios, and we may need to accept only the trends of their variations. Landier and Thesmar (2008) found a positive correlation between optimism and the use of short-term debt using a data set of French entrepreneurs, but they did not consider the effects on long-term debt. This implication might need to be checked from another empirical test.

Another interesting result is that the equity value ratio declines with the degree of optimism, which is consistent with the empirical result of Dittmar and Thakor (2007) that firms with high heterogeneity have an increase in leverage in the year of the security issuance.

To sum up our discussion on the effects of managerial optimism, we look at the results about the effects of managerial risk aversion  $\lambda$  in Figures 3 and 4, which may tell us the implication of managerial optimism. In this model, the firm's dynamic capital structure is optimally determined in a way of setting optimal risk-sharing between the parties with differing attitudes towards the risks and of providing incentives for co-investments of human and physical capital by them. Since the manager's risk aversion obviously reflects the deteriorating effect on the agency conflicts in

this model, by observing exactly opposite trends compared to the variations of capital structure and contract variables with managerial risk aversion, we may argue that managerial optimism has beneficial effects on the manager-shareholders agency relationship.

We now move on to the discussion on the effects of two distinctive risk factors of the firm's earnings. In Figures 5 and 6, the initial transient and intrinsic risks have differing effects on the long-term debt and short-term debt ratios. Note that we smooth the variations of these values using a linear trend. There are two issues regarding these risks. On the one hand, both risks increase the costs of providing incentives to the manager, as reflected in the downward trends of the manager's pay-performance sensitivity and effort. Since these changes lower the manager's continuation value relative to her initial payoff, she will choose greater long-term debt at the margin in order to exploit debt tax shields. According to Giat et al. (2009), on the other hand, these two risks also affect the variance of the evolution of the mean assessments of the core output in opposite ways, as shown in (26). The variance increases monotonically with the initial transient risk, but decreases with the intrinsic risk. Hence, an increase in the transient (intrinsic) risk implies that intermediate signals about the core output from the realization of earnings are relatively more (less) informative and thus that the manager's option value of continuing the relationship increases (decreases) with the risk, which implies that the manager will choose lower (greater) long-term debt.

From these underlying effects, we can explain the decreasing trend of long-term debt variations with respect to the initial transient risk by the positive effect of the risk on the option value of continuing the relationship that dominates its negative effect on the costs of risk-sharing. However, long-term debt increases with the intrinsic risk in both aspects. Although an increase in either risk lowers the manager's pay-performance sensitivity, since the long-term debt coupon itself, as explained above, affects the short-term debt, the short-term debt ratio increases with the initial transient risk, but decreases with the intrinsic risk.

Lastly, it must be noted that the equity value ratio increases with the intrinsic risk, while it barely responds to the initial transient risk. The effect of the intrinsic risk is consistent with the capital structure literature that have mainly focused on the permanent risk and suggested the leverage ratio as a decreasing function of the volatility of earnings, but the transitory risk has little effect on the leverage ratio in our results, which contrasts to the result of Gorbenko and Strebulaev (2008) that firms show more conservative financial response when transitory shocks are present.

### 3.2.3 Analysis of Firms Going Public

As mentioned, we finally analyze the case of initial public offerings. Although we have observed the significant levels of manager optimism and transient risk about the core output for general firms, new startups seem to be better explained by these features. Selecting a sample of firms that complete the offering over the period of 1993-2003, including the dot-com bubble, we obtain their observed median values of the statistics, which are reported with the predicted values of the statistics as a result of the calibration of the model in Table 3. Compared to the observed values for general firms in Table 1, lower debt values and higher firm value of this sample firms have been widely accepted in the literature. The estimates of inside equity stake represent the median stake of officers and directors as a group and that of the CEO, respectively, which come from different studies. Even only with the CEO ownership stake, the firms that provide the offering have a higher inside equity stake. Finally, slightly higher investment ratio and lower earnings ratios are also not surprising in that most of the firms that go public are newly formed innovative firms that require more investment projects and do not have stable earnings flows yet.

With these noticeable differences in mind, we may anticipate differing baseline parameter values for these firms from those for general firms. In Table 4, the baseline values of the belief parameters that we are interested in turn out to be higher, as expected. From higher  $\Delta_0$  relative to  $\mu_0^S$ , we can say that entrepreneurs have greater relative optimism than general managers have. While the difference in the intrinsic earnings risk that is directly computed from the data is negligible, IPO firms have a higher level of the initial transient risk about the project quality. Moreover, it would be also interesting to compare the values of technology and preference parameters. The higher total factor productivity reasonably characterizes high growth businesses, and the lower disutility of effort and higher risk aversion parameters also reflect the characteristics of entrepreneurs well who are willing to take upon herself or himself new innovative projects, but may be less diversified (more firm-specific).

## 4 Conclusions

We develop a dynamic structural model to examine the effects of heterogeneous beliefs and agency conflicts on corporate capital structure. In the dynamic capital structure framework incorporating taxes, bankruptcy costs, managerial discretion in financing and effort, and endogenous bankruptcy, we add two key features: (i) heterogeneous beliefs about the core output in the firm's earnings and

differing risk attitudes and (ii) dynamic actions by both parties, which are the manager's effort and shareholders's capital investments. We derive the manager's optimal dynamic contract and financing choice and implement the agency contract through financial securities, which leads to the firm's dynamic capital structure and therefore allows us to obtain the capital structure implications.

Through the analysis of the calibrated model, we derive new testable implications that link manager optimism and underlying project characteristics to capital structure: (i) Long-term debt declines with managerial optimism, whereas short-term debt increases with it; (ii) Long-term debt (Short-term debt) decreases (increases) with the transitory earnings risk, but increases (decreases) with intrinsic earnings risk; (iii) Equity value is negatively affected by managerial optimism, but positively affected by intrinsic earnings risk. Our analysis suggests that managerial optimism and transitory and permanent earnings risks are important determinants of firms' optimal capital structure.

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**Table 1: Observed and Predicted Statistics for General Firms.** The table displays the observed values of the statistics that are used to calibrate the model and their predicted values from the model. We scale all variables, except for the inside equity stake, by the asset value. The observed inside equity stake is consistent with the estimate of Holderness et al (1999), but the other observed values are based on a sample of general firms from the annual Compustat industrial files over the period of 1993-2003. The predicted statistics are estimated from the model, which characterizes the manager’s optimal financing and contracting choices and therefore the firm’s dynamic capital structure, using a discretization approach.

	Observed	Predicted
Long-term Debt Ratio	0.0949	0.0977
Short-term Debt Ratio	-0.0738	-0.0753
Firm Value Ratio	1.1285	1.1481
Inside Equity Stake	0.1440	0.1442
Investment Ratio	0.0436	0.0039
EBITM Ratio	0.0953	0.0789
Net Income Ratio	0.0182	0.0494

**Table 2: Baseline Parameter Values for General Firms.** The table reports the baseline parameter values obtained by the model calibration for general firms. The parameters are classified into the three categories: "Technology" parameters including the total-factor productivity,  $A$ , and the intrinsic risk,  $s$ , "Belief" parameters consisting of the mean assessment of the core output by shareholders,  $\mu_0^{Pr}$ , the degree of heterogeneity in beliefs,  $\Delta_0$ , and the initial transient risk,  $\sigma_0$ , "Preference" parameters incorporating the manager’s risk aversion parameter,  $\lambda$ , and disutility of effort parameters,  $\kappa$  and  $\gamma/\beta$ .

Technology Parameters		Belief Parameters			Preference Parameters		
$A$	$s$	$\mu_0^S$	$\Delta_0$	$\sigma_0$	$\lambda$	$\kappa$	$\gamma/\beta$
21.89	39.31	77.88	124.61	59.88	0.22	80.99	7.33

**Table 3: Observed and Predicted Statistics for Firms that Go Public.** The table shows the observed values of the statistics used to calibrate the model and their predicted values from the model. While the observed inside equity stake is consistent with the estimate of Mikkelsen, Partch, and Shah (1997), the other observed values are based on a sample of the firms with the initial public offering over the period of 1993-2003 from the annual Compustat industrial files.

	Observed	Predicted
Long-term Debt Ratio	0.0162	0.0206
Short-term Debt Ratio	-0.3315	-0.3299
Firm Value Ratio	1.9873	1.9869
Inside Equity Stake	0.1590	0.1479
Investment Ratio	0.0467	0.0189
EBITM Ratio	0.0697	0.1166
Net Income Ratio	0.0096	0.0544

Table 4: **Baseline Parameter Values for Firms that Go Public.** The table displays the baseline parameter values obtained by the model calibration for the firms that go public.

Technology Parameters		Belief Parameters			Preference Parameters		
$A$	$s$	$\mu_0^S$	$\Delta_0$	$\sigma_0$	$\lambda$	$\kappa$	$\gamma/\beta$
64.83	42.73	108.60	231.52	86.90	0.42	33.77	2.88

Figure 1: **The Effects of Managerial Optimism (variations of contract variables)**

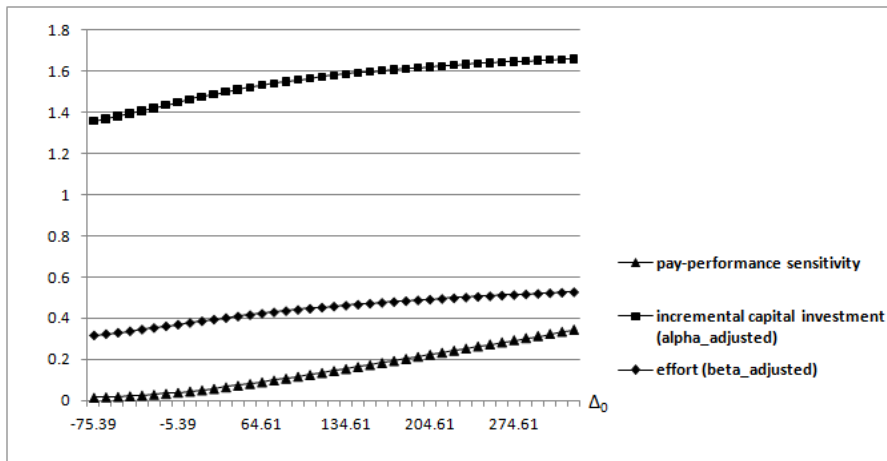


Figure 2: **The Effects of Managerial Optimism (variations of capital structure variables)**

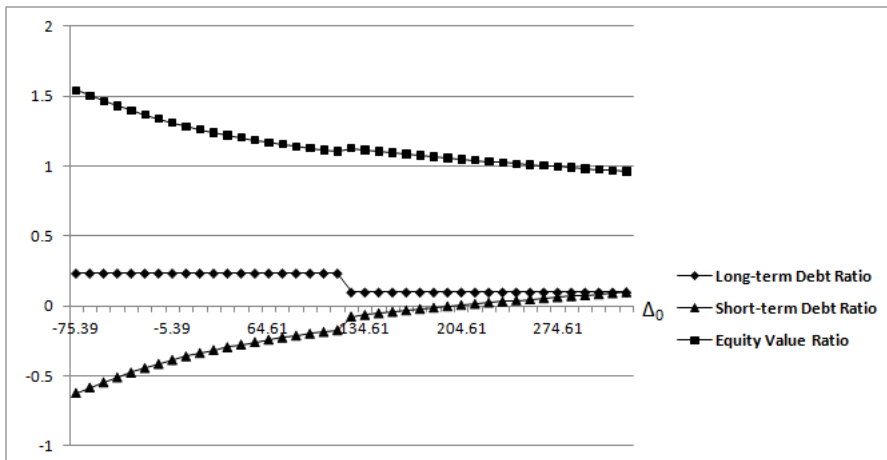


Figure 3: The Effects of Managerial Risk Aversion (variations of contract variables)

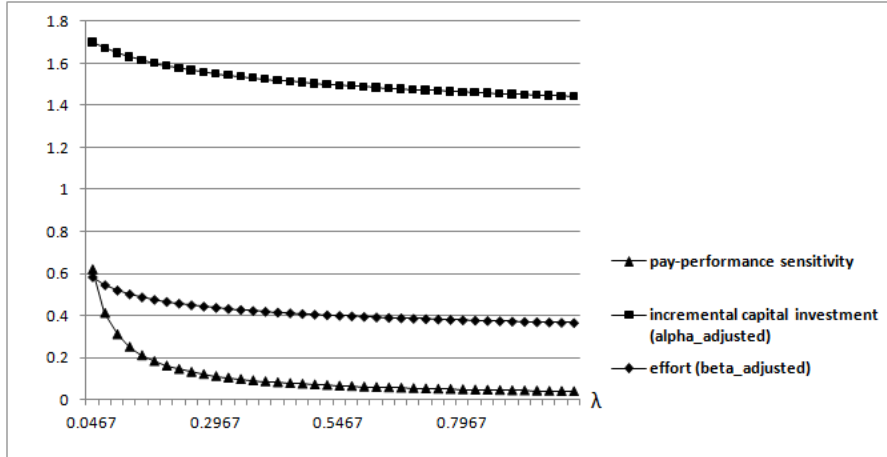


Figure 4: The Effects of Managerial Risk Aversion (variations of capital structure variables)

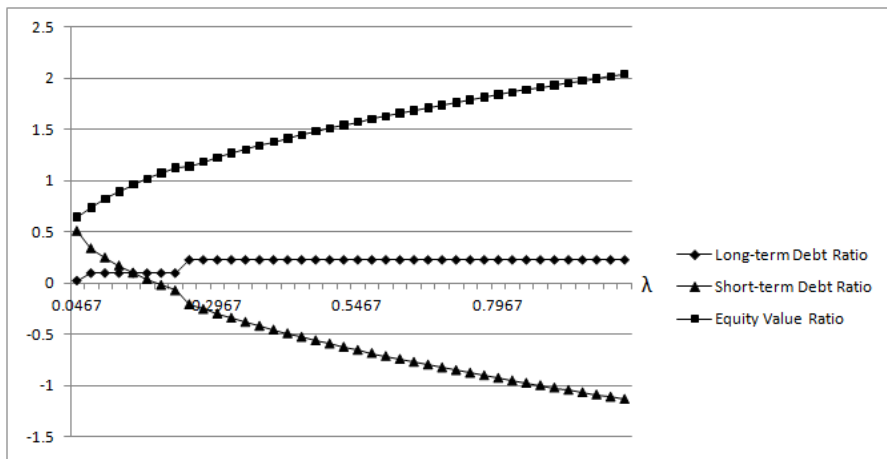


Figure 5: The Effects of Initial Transient Risk

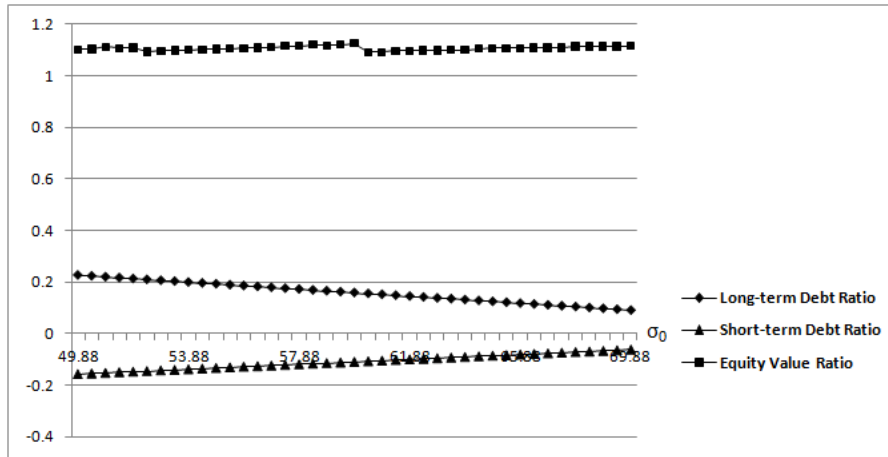


Figure 6: The Effects of Intrinsic Risk

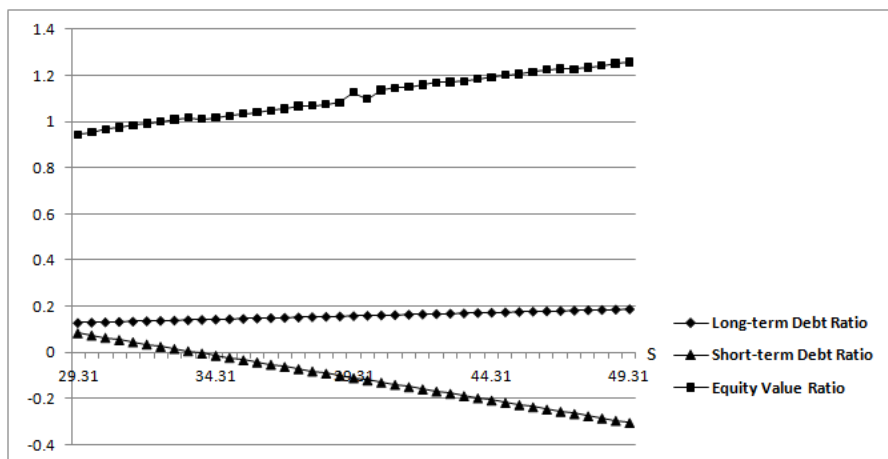


Figure 7: The Effects of Managerial Ability

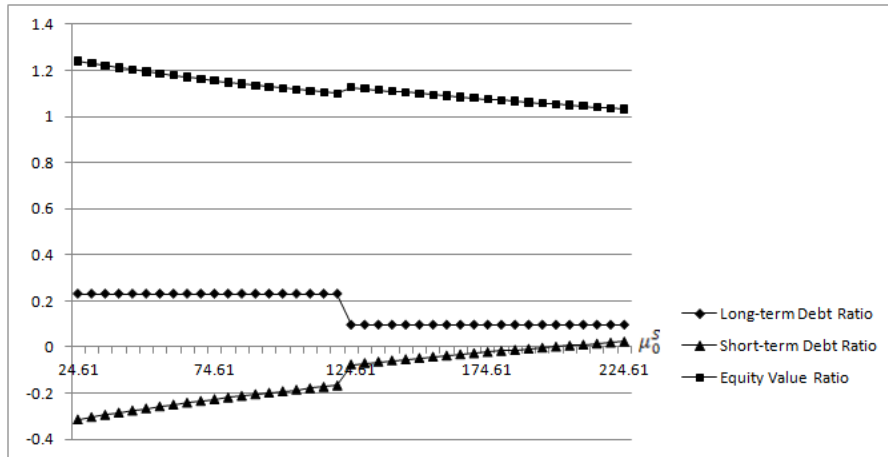


Figure 8: The Effects of Managerial Disutility of Effort

