Personal Social Security Accounts: Just How Important Are Annuity Markets? *

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Abstract

Are private annuity markets really a good thing? Although it sounds counterintuitive, the answer may be "no" in an overlapping-generations economy, where household's saving, bequests, tax revenue, and tax rates are endogenously determined. This paper constructs dynamic general equilibrium (and partial equilibrium) OLG models to analyze the macroeconomic and welfare effects of private annuity markets. Solving models for equilibrium transition paths, we find the introduction of perfect annuity markets always reduces household saving and national output; it generates efficiency gains in an economy without a progressive income tax when the inheritance of wealth is stochastic; however, the efficiency effect is small and ambiguous in an economy with an income tax; and the marginal efficiency effect of annuity markets small and possibly negative when Social Security pension is privatized.

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1 Introduction

In our past research, using an overlapping-generations model (OLG) with wage and longevity uncertainty, we show that a 50% partial privatization of Social Security leads to sizeable efficiency gains in the presence of insurable wage shocks by improving labor supply incentives. But when, more realistically, wage shocks are not insurable, efficiency losses emerge from privatization despite the fact that this reform produces the increase in major macroeconomic variables —capital stock, labor supply, and output—along the equilibrium transition path and in the long run.

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A question that often arises in the debate over Social Security reform is whether private markets would really allow people with personal accounts to receive the type of annuity protection provided under the traditional Social Security system. The empirical evidence is fairly convincing that current private annuity markets are not perfect (e.g., Mitchell, *et al.*, 1999), although Social Security itself might actually be a cause rather than a solution (Abel, 1986). An open question is how private annuity markets will endogenously respond to personal accounts. Answering that question requires the use of a dynamic general-equilibrium OLG model that involves considerable complexity.

A logical first step is to examine whether the supposition underlying the debate over personal accounts and private annuity markets is actually true. In particular, is it correct that private annuity markets are really a good thing? The apparent answer seems "yes" because private annuities help pool idiosyncratic longevity risks. However, as radical and counterintuitive as it might sound at first glance, the actual answer is maybe "no"—at least once it is recognized that the government must rely on second-best tax instruments in order to afford public consumption (e.g., education, roads, defense, *etc.*) and redistribution.

Most theoretical analysis of annuity markets, starting with the seminal work by Yaari (1965), has ignored the interaction between annuity markets and the government's entire budget constraint. Even studies focusing on Social Security and annuity markets have typically treated the "rest of government" as irrelevant to the discussion. The OLG model that we have developed shows that these interactions are actually quite important, in fact, and of first-order magnitude. As it turns out, these interactions extend to the annuity debate as well.

More specifically, in a world with second-best tax instrument, as first developed in the work by James Mirrlees (1971), the government must tax sources of income, such as labor and capital flows, rather than "intrinsic" metrics. Labor and capital income taxes, in turn, cause people to distort their labor and savings. A fact that has not been appreciated thus far in the literature is that insurance instruments, including annuity products, also allow people to reduce their saving and labor supply, thereby escaping some taxation.

In the case of annuities, this fact is true even if the annuity products do not contain a direct tax advantage, i.e., accrued returns are taxable just like any other capital income. Intuitively, annuity products allow people to reduce their level of precautionary saving thereby reducing their tax bill. A smaller tax bill paid one person requires a larger tax rate for everyone. In essence, private annuity markets create a negative non-pecuniary externality *vis-à-vis* the tax system. In other words, insurance products help promote "free-riding."

The open question is whether the direct insurance value of annuity products is more important or less important than the indirect value of the free-riding problem created by annuity products. We conducted some preliminary simulations with a simplified version of our OLG and found many cases with reasonable calibrations to the U.S. economy in which the indirect free-riding effect was actually more important the direct insurance effect even if the annuity products had no direct tax advantage. In other words, rather than subsidizing annuity products as in the United States, the optimal policy might actually be to tax them. For the purpose of Social Security reform, the debate over private annuity markets might be very misplaced: privatization might actually work better if private annuity markets are difficult to form after privatization.

At first glance, it might appear that the dominance of the indirect effect is actually due to a flawed tax system, in particular, a positive tax on capital. Indeed, the seminal papers by Judd (1985) and Chamley (1986) have argued that taxes on capital income should be zero in the long run. However, this result makes several assumptions including households being infinitely-lived and no idiosyncratic risk. Relaxing either of these strong assumptions produces a positive optimal tax rate on capital income in steady state (e.g., Aiyagari, 1995). Our model relaxes both assumptions.

The goal of this paper is to examine whether our result—in which the free-riding effect produced by annuities dominates the insurance effect—holds in a more complicated and realistic version of our OLG model. Moreover, we would like to make some headway on the theory side, although theoretical advances are always a challenge in a model with precautionary savings. Not only could our results could turn the annuity and social security debate on its head, they also point to the importance of evaluating government policies in a holistic framework instead of focusing on just one policy at a time.

The Outline of This Paper.

We first construct a simple two-period OLG model to show analytically that introducing perfect annuity markets may or may not improve the efficiency of the economy. When we assume a 0% interest rate (i.e., no capital income) and deterministic labor income, introducing perfect annuity markets improves the efficiency both in the short run and in the long run. When we assume a positive interest rate so that the saving decision would affect the lifetime income, however, introducing perfect annuity markets may or may not improve the welfare of the future households, depending on the parameter values, although it always improve the welfare of the initial young households in our two-period setting. The overall efficiency gain or loss depends on the survival rate at the end of the first age period, and the efficiency gain is smaller (loss is larger) if the interest rate is higher.

To evaluate the effect of perfect annuity markets more precisely, we next introduce a deterministic 90-period OLG model with representative households. Using this model, we conduct two types of policy experiments—introducing perfect annuity markets and privatizing pay-as-you-go Social Security—

under several different assumptions. Regarding the first set of policy experiments, introducing perfect annuity markets, we assume that the economy is either closed or small open, that accidental bequests in the baseline economy are distributed either deterministically (uniformly) or stochastically, and that the baseline economy is either equipped with a progressive income tax or not.

The introduction of perfect annuity markets reduces national wealth and output (GNP). In our experiments, we assume to change the flat labor and capital income tax rate to balance the government budget. In a 90-period deterministic OLG economy without a progressive income tax, introducing perfect annuity markets reduces the overall efficiency slightly, equivalent with one time wealth loss between $1,100 and $1,300 per future household, when accidental bequests are redistributed uniformly and deterministically to all working-age households before the policy change. If we assume that accidental bequests are inherited stochastically, however, introducing perfect annuity markets generates the efficiency gain between $4,900 and $5,200 per future household by reducing the overall risk in the economy.

When we assume a progressive income tax in the baseline economy, however, the efficiency effect of perfect annuity markets is small and ambiguous. Under the stochastic inheritance assumption of the baseline economy, the efficiency gain is reduced to $700 in a closed economy and -$1,100 in a small open economy. Overall, in a realistic economy with stochastic inheritance and a progressive income tax, the effect of introducing perfect annuity markets is small and ambiguous.

How does the effect of privatizing social security differ with and without perfect annuity markets in the pre-reform economy? In the economy without a progressive income tax, if perfect annuity markets are available both in the baseline economy and in the post-reform economy, a 50% privatization of Social Security improves the overall efficacy slightly by $100 per future household in a closed economy, but it generates the efficiency loss of $3,700 in a small open economy. If we assume an economy without perfect annuity markets and assume uniform and deterministic inheritance, a partial privatization of Social Security generates efficiency losses between $5,600 and $10,700. The efficiency losses are even larger—between $8,700 and $10,400—in the economy with stochastic inheritance. These results imply that the annuity aspect of current Social Security pension might be important for the economy.

If we assume the economy with a progressive income tax, however, the efficiency effect of privatizing Social Security is significantly different. The privatization improves the efficiency of the economy significantly. In the economy with perfect annuity markets, the overall efficiency gain is between $24,100 and $32,600 per future household. In the economy without perfect annuity markets and with deterministic inheritance, the efficiency gain is even larger, and it is between $37,100 and

$41,500, because the privatization increases not only lifecycle savings of households but also precautionary savings. In the absence of annuity markets, the interest-rate elasticity of saving is lower due to a larger precautionary saving motive, thus, the distortion from the original capital income tax would be smaller. In addition, a larger increase in saving and output generates higher tax revenue, which allows the government to reduce labor income and capital income tax rates.

When we assume stochastic inheritance of accidental bequests, the efficiency gains are smaller than the those in the economy with the uniform redistribution. The efficiency gain is between $29,500 and $30,600 per future household, because households would be exposed to a larger uncertainty of inheritance.

(In the next version of this paper, we also use the stochastic and heterogeneous-agent OLG model to do the same set of policy experiments to evaluate the efficiency gains from introducing perfect annuity markets and privatizing Social Security.)

The rest of the paper is as follows: Section 2 describes a simple 2-period OLG economy and shows that the introduction of perfect annuity markets does not necessarily generate efficiency gains; Section 3 describes a large-scale OLG model with and without idiosyncratic working ability shocks as well as mortality shocks; Section 4 explains the calibration of the large-scale OLG model; Section 5 presents the macroeconomic and efficiency effects of introducing perfect annuity markets; Section 6 investigates the effects of Social Security privatization with and without perfect annuity markets; and Section 7 concludes the paper.

2 Simple 2-Period OLG Examples

There are a large number of *ex ante* identical households in the economy. Each household lives either one or two periods and, in each period, there are two generations—young and old—in the economy. The population of young households is normalized to unity. There is no population growth. Let c_1 and c_2 be the consumption in period 1 and in period 2, let ϕ be the survival rate of a young household at the end of the first period, let w be the labor income of a young household, let r be the interest rate, and let b be the accidental bequest each young household received at the beginning of its first age period. In this section, for simplicity, any remaining wealth left by deceased households is distributed uniformly to young households.

2.1 An Economy without Annuity Markets

Suppose that the lifetime utility function is log-linear with the subjective time discount factor β. Then, a household's utility maximization problem can be written as

$$\max_{c_1,c_2} E\,u(c_1,c_2) = \ln c_1 + \beta\phi \ln c_2$$

subject to the budget constraint

$$c_1 + \frac{c_2}{1+r} \leq w + (1+r)\,b.$$

Solving this problem, we get

$$c_1^* = \frac{w + (1+r)\,b}{1+\beta\phi}, \quad \frac{c_2^*}{1+r} = \frac{\beta\phi\,[w + (1+r)\,b]}{1+\beta\phi} = s^*,$$

where s is the saving at the end of first period.

The steady-state condition is

$$b = (1-\phi)\,s^* = \frac{\beta\phi\,(1-\phi)\,[w + (1+r)\,b]}{1+\beta\phi},$$

since there is no population growth. Solving this equation for b, we get

$$b = \frac{\beta\phi\,(1-\phi)\,w}{1+\beta\phi^2 - \beta\phi\,(1-\phi)\,r}.$$

The steady-state consumption schedule is

$$c_1^* = \frac{1}{1+\beta\phi^2 - \beta\phi\,(1-\phi)\,r}w, \quad c_2^* = \frac{(1+r)\,\beta\phi}{1+\beta\phi^2 - \beta\phi\,(1-\phi)\,r}w.$$

2.2 The Introduction of Annuity Markets

Suppose that perfect annuity markets are introduced to the economy. The actuarially fair price of the annuity is equal to the survival rate at the end of the first period. Since it is optimal for a household to fully annuitize its wealth at the end of the first period, the household's problem becomes

$$\max_{c_1,c_2} E\,u(c_1,c_2) = \ln c_1 + \beta\phi \ln c_2$$

subject to

$$c_1 + \frac{\phi c_2}{1+r} \leq w.$$

Solving this problem, the steady-state consumption schedule with annuity markets is

$$c_1^{**} = \frac{w}{1+\beta\phi}, \quad \frac{c_2^{**}}{1+r} = \frac{\beta w}{1+\beta\phi}.$$

The transition path of this 2-period OLG economy is simple. When perfect annuity markets are introduced, the economy reaches to the new steady state in two periods, i.e., there is only one transition generation—initial young households—who receive accidental bequests from the previous generation and are able to annuitize their own saving. The problem of an initial young household is

$$\max_{c_1, c_2} E\, u\left(c_1, c_2\right) = \ln c_1 + \beta\phi \ln c_2$$

subject to

$$c_1 + \frac{\phi c_2}{1+r} \leq w + (1+r)\, b.$$

Solving this problem, the consumption schedule in the transition is

$$c_1^{**} = \frac{w + (1+r)\, b}{1+\beta\phi}, \quad \frac{c_2^{**}}{1+r} = \frac{\beta\left[w + (1+r)\, b\right]}{1+\beta\phi}.$$

2.3 Numerical Example (1)

Let's first assume that both the interest rate and the time discount rate are zero. Then, the lifetime income of a household is not affected by the household's consumption-saving decision. The household's consumption schedule in the absence of annuity markets is simplified to

$$c_1^{*} = \frac{w}{1+\phi^2}, \quad c_2^{*} = \frac{\phi w}{1+\phi^2}.$$

Suppose that a perfect annuity market is introduced to this economy. The household's consumption schedule becomes

$$c_1^{**} = c_2^{**} = \frac{w}{1+\phi}.$$

Table 1 shows the long-run effect of introducing perfect annuity market to this simple 2-period

OLG economy. The wage rate is assumed to be unity. In the absence of annuity markets, consumption in the first age-period tends to be higher than that in the second age-period. If perfect annuity markets are present, households can perfectly smooth out their consumption. The expected lifetime utility in the economy with annuity markets, Eu^{**}, is higher than the expected utility in the economy without annuity markets, Eu^{*}, for any survival rate ϕ between 0 and 1. We can numerically verify that

$$E\,u\left(c_1^{**}, c_2^{**}\right) - E\,u\left(c_1^{*}, c_2^{*}\right) = (1+\phi)\ln\frac{1+\phi^2}{1+\phi} - \phi\ln\phi > 0 \quad \forall\phi \in (0,1)\,,$$

i.e., the introduction of perfect annuity markets would generate the efficiency gain in the long run. The initial young households, which are the only transition households, would clearly be better off because, in our assumption, they receive accidental bequests at the beginning of the first period and they are also benefitted from perfect annuity markets newly introduced. Accordingly, all households in the equilibrium transition path would be better off if perfect annuity markets were introduced. Since introducing perfect annuity markets is Pareto improving, it would also generate an efficiency gain in the economy.

2.4 Numerical Example (2)

When the interest rate is positive, however, the welfare effect of introducing perfect annuity markets is not obvious. Table 2 shows the effect of introducing perfect annuity markets when $w = 1$, $r = 0.5$, and $\beta = 1/\left(1+r\right) = 2/3$. Under this assumption, the lifetime utility of households in the steady-state economy with perfect annuity markets is higher that in the economy without annuity markets when the survival rate ϕ is between 0.0 and 0.33. However, the lifetime utility in the economy with annuity markets is lower when ϕ is between 0.33 and 1.0. That is, the long-run welfare gain is positive when the survival rate ϕ is relatively small but negative when ϕ is relatively large. As before, the transition households—the initial young households—will be better off for all ϕ between 0 and 1. Thus, the overall welfare gain or efficiency gain is ambiguous when ϕ is relatively large.

Table 3 shows the effect of introducing perfect annuity markets with parameters $w = 1$, $r = 1.0$, and $\beta = 0.5$. The long-run welfare gain from the introduction of annuity markets is smaller when we assume a higher interest rate and higher time discount rate. The lifetime utility of households would be increased when ϕ is between 0.0 and 0.17, but it would be smaller when ϕ is between 0.17 and 1.0.

To check whether the overall efficiency gain from introducing perfect annuity markets is positive or negative, we next calculate the welfare changes by the (negative of) compensating variation in wealth—how much one-time wealth transfer is needed for each household to make her as better off after introducing perfect annuity markets as she were in the baseline economy without annuity markets. Suppose that perfect annuity markets are introduced at the beginning of period 1 and $tr_{CV,t}$ denotes the one-time wealth transfer made in period t needed to recover the lifetime utility of young households in t. (Note that the lifetime utlity of initial old households do not change in this simple economy.) The overall efficiency gain is calculated as the negative of present discount value of the compensating variations in wealth, i.e.,

$$-\sum_{t=1}^{\infty} \frac{tr_{CV,t}}{(1+r)^{t-1}} = -tr_{CV,1} - \frac{tr_{CV,\infty}}{r},$$

since $tr_{CV,0} = 0$ and $tr_{CV,2} = tr_{CV,3} = \cdots = tr_{CV,\infty}$ in this simple 2-period model economy. (Later in this paper, we will introduce a fictitious government institution called the Lump-Sum Redistribution Authority to evaluate the general equilibrium version of the compensating variations in wealth.)

Table 4 shows that the introduction of perfect annuity markets would generate efficiency gains or losses depending on the survival rate at the end of the first age period. Under the assumption of $r = 0.5$ and $\beta = 2/3$, introducing perfect annuity markets would generate efficiency gains if ϕ is 0.63 or lower, but it would generate efficiency losses if ϕ is higher than 0.63. The efficiency gain tends to be smaller when the size of capital income relative to labor income is larger. When $r = 1.0$ and $\beta = 0.5$, introducing perfect annuity markets would generate efficiency loss when ϕ is higher than 0.35.

3 A Large-Scale OLG Model Economy

In this section, we introduce a 90-period dynamic general equilibrium overlapping-generations model to examine the macroeconomic and efficiency effects of perfect annuity markets. The model is a slightly simpler version of the model in Nishiyama and Smetters (2005). There are three sectors in the economy: heterogeneous households with elastic labor supply, a competitive firm with constant-returns-to-scale production technology, and a government. Time is discrete and the unit of time period is a year.

3.1 The Household

Households are heterogeneous with respect to age i, working ability e (measured by hourly wages), and beginning-of-period wealth a. Each year, a large number of new households of age 20 enter the economy. Population grows at a constant rate ν. A household of age i observes an idiosyncratic working ability shock e at the beginning of each year and chooses its optimal consumption c, working hours h, and end-of-period wealth holding a', taking as given future factor prices, tax rates and pension benefits, the probability distribution of inheritance. At the end of each year, a fraction of households die according to standard mortality rates, and no one lives beyond age 109. For simplicity, all households represent two-earner married couples of the same age.

Let $\mathbf{s}$ denote the individual state of a household,

$$\mathbf{s} = (i, e, a),$$

where $i \in I = \{20, ..., 109\}$ is the household's age, $e \in E = [e_{\min}, e_{\max}]$ is its age-dependent working ability (the hourly wage), and $a \in A = [a_{\min}, a_{\max}]$ is its beginning-of-period wealth.

Let $\mathbf{S}_t$ denote the state of the economy at the beginning of year t,

$$\mathbf{S}_t = (x_t(\mathbf{s}), W_{LS,t}, W_{G,t}),$$

where $x_t(\mathbf{s})$ is the joint distribution of households, $W_{LS,t}$ is the beginning-of-period net wealth held by the Lump-Sum Redistribution Authority (LSRA) described below, and $W_{G,t}$ is the net wealth of the rest of the government.

Let $\mathbf{\Psi}_t$ denote the government policy schedule known at the beginning of year t,

$$\mathbf{\Psi}_t = \{W_{LS,s+1}, W_{G,s+1}, C_{G,s}, \tau_{I,s}(.), \tau_{P,s}(.), tr_{SS,s}, tr_{LS,s}(\mathbf{s})\}_{s=t}^{\infty},$$

where $C_{G,s}$ is government consumption, $\tau_{I,s}(.)$ is an income tax function, $\tau_{P,s}(.)$ is a payroll tax function for Social Security (OASDI), $tr_{SS,s}$ is a Social Security benefit, and $tr_{LS,s}(\mathbf{s})$ is an LSRA wealth redistribution function.

The household's problem is

$$v(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t) = \max_{c,h} u(c, h) + \beta(1+\mu)^{\alpha(1-\gamma)} \phi_i E \left[v\left(\mathbf{s}', \mathbf{S}_{t+1}; \mathbf{\Psi}_{t+1}\right) | e \right] \tag{1}$$

subject to

$$\begin{aligned} a' = \frac{1}{1+\mu} \{ & w_t eh + (1+r_t)(a + tr_{LS,t}(\mathbf{s})) - \tau_{I,t}(w_t eh + r_t(a + tr_{LS,t}(\mathbf{s}))) \\ & - \tau_{P,t}(w_t eh) + tr_{SS,t} - c\} \geq a'_{\min,t}(\mathbf{s}), \\ & a = 0 \text{ if } i = 20, \quad a \geq 0 \text{ if } i \geq 65, \end{aligned} \tag{2}$$

where the utility function, $u(.)$, takes the Cobb-Douglas form nested within a time-separable isoelastic specification,

$$u(c,h) = \frac{\{(c^{\alpha}(h_{\max} - h)^{1-\alpha}\}^{1-\gamma}}{1-\gamma}; \tag{3}$$

γ is the coefficient of relative risk aversion and $h_{\max}$ is the maximum working hours. Wages follow a Markov process. The constant β is the rate of time preference, ϕ_i is the survival rate at the end of age i, w_t is the wage rate per efficiency unit of labor, and r_t is the rate of return to capital. Individual variables are normalized by the labor augmenting productivity growth rate μ.

The state-contingent minimum level of end-of-period wealth, $a'_{\min,t}(\mathbf{s})$, is a natural borrowing limit, that is, the debt is repayable even if the household receives the worst possible shocks in future working abilities. At the beginning of the next period, the state of this household when private annuity markets do not exist becomes

$$\mathbf{s}' = (i+1, e', a' + q_t), \tag{4}$$

where q_t denotes accidental bequests that a household receives at the end of the period. In the presence of perfect annuity markets, the household's state in the next period is instead

$$\mathbf{s}' = (i+1, e', a'/\phi_i). \tag{5}$$

Let $x_t(\mathbf{s})$ denote the population distribution of households, and let $X_t(\mathbf{s})$ be the corresponding cumulative distribution. The distribution of households is adjusted by the steady-state population growth rate, ν. The population of age 20 households is normalized to unity, that is,

$$\int_E \mathrm{d}X_t(20, e, 0) = 1.$$

Let $\mathbf{1}_{[a=y]}$ be an indicator function that returns 1 if $a = y$ and 0 if $a \neq y$. Then, the law of motion of the measure of households is

$$x_{t+1}(\mathbf{s}') = \frac{\phi_i}{1+\nu} \int_{E \times A} \mathbf{1}_{[a'=a'(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t)+q_t]} \pi_{i,i+1}(e'|e) \mathrm{d}X_t(\mathbf{s}),$$

where $\pi_{i,i+1}$ denotes the transition probability of working ability from age i to age $i+1$.

Accidental bequests can be simply distributed equally and deterministically across all surviving working-age households in the model or, more realistically, accidental bequests can be distributed randomly to surviving working-age households. In the latter strategy, each household receives a bequest

q_t with constant probability η:

$$q_t = \frac{\sum_{i=20}^{109}(1-\phi_i)\int_{E\times A} a'(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t)\mathrm{d}X_t(\mathbf{s})}{\sum_{i=20}^{109}(1-\phi_i)\int_{E\times A} \mathrm{d}X_t(\mathbf{s})},$$

$$\eta = \frac{\sum_{i=20}^{109}(1-\phi_i)\int_{E\times A} \mathrm{d}X_t(\mathbf{s})}{\sum_{i=20}^{64}\phi_i\int_{E\times A} \mathrm{d}X_t(\mathbf{s})}.$$

where q_t is the average wealth left by deceased households, and η is the ratio of deceased household to the surviving working-age households.

3.2 Government

Government tax revenue consists of a progressive income tax $T_{I,t}$ and a payroll tax for Social Security (OASDI) $T_{P,t}$. These revenues are

$$T_{I,t} = \sum_{i=20}^{109}\int_{E\times A} \tau_{I,t}(w_t e h(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t) + r_t(a + tr_{LS,t}(\mathbf{s})))\mathrm{d}X_t(\mathbf{s}), \tag{6}$$

$$T_{P,t} = \sum_{i=20}^{109}\int_{E\times A} \tau_{P,t}(w_t e h(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t))\mathrm{d}X_t(\mathbf{s}). \tag{7}$$

Social Security (OASDI) benefit expenditure $Tr_{SS,t}$ is

$$Tr_{SS,t} = tr_{SS,t}\sum_{i=65}^{109}\int_{E\times A} \mathrm{d}X_t(\mathbf{s}). \tag{8}$$

The law of motion of the government wealth (normalized by productivity growth and population growth) is

$$W_{G,t+1} = \frac{1}{(1+\mu)(1+\nu)}\{(1+r_t)W_{G,t} + T_{I,t} + T_{P,t} - Tr_{SS,t} - C_{G,t}\}, \tag{9}$$

where $C_{G,t}$ is government consumption.

3.3 The Evaluation of Efficiency Gains

We follow Auerbach and Kotlikoff (1987) by measuring efficiency gains from a policy change using a Lump-Sum Redistribution Authority that compensates households who would otherwise lose from reform. The LSRA is simply a hypothetical mechanism that allows us to measure the standard Hicksian efficiency gains in general equilibrium associated with privatization. A policy reform that increases Hicksian efficiency is *potentially* Pareto improving whereas a reform that reduces efficiency cannot be Pareto improving.

To see how the LSRA works, suppose that a new policy is announced at the beginning of period 1. First, the LSRA makes a lump-sum compensating variation transfer or tax, $tr_{CV,1}(\mathbf{s})$, to each *living* household of age i in order to return its expected remaining lifetime utility at state $\mathbf{s}$ to its pre-reform level in the baseline (pre-reform) economy. Next, the LSRA makes a lump-sum transfer or tax, $tr_{CV,t}(\mathbf{s})$, to *each future* household (born in periods 2, 3, ...) to make it as well off as in the baseline economy, conditional on its initial state at age 20. Thus far, however, the net present value of these taxes and transfers across living and future households will generally not sum to zero. So, finally, the LSRA makes an additional lump-sum transfer (tax), Δtr, to each *future* household so that the net present value across all transfers is zero. For illustrative purposes, we assume, like Auerbach and Kotlikoff, that these additional transfers are uniform across future generations on a growth-adjusted basis. The lump-sum transfers made by the LSRA, therefore, are

$$tr_{LS,t}(\mathbf{s}) = \begin{cases} tr_{CV,t}(\mathbf{s}) & \text{if } t = 1 \\ tr_{CV,t}(\mathbf{s}) + \Delta tr & \text{if } t > 1 \text{ and } i = 20 \\ 0 & \text{otherwise} \end{cases} . \tag{10}$$

If $\Delta tr > 0$ then privatization has produced net new resources and so we say that this reform "increases efficiency." Conversely, if $\Delta tr < 0$ then privatization "reduces efficiency."

The aggregate net lump-sum transfers / taxes to living households at time t, $Tr_{LS,t}$, is

$$Tr_{LS,t} = \sum_{i=20}^{109} \int_{E \times A \times B} tr_{LS,t}(\mathbf{s}) \mathrm{d}X_t(\mathbf{s}). \tag{11}$$

The law of motion of the LSRA wealth (normalized by productivity growth and population growth), therefore, is

$$W_{LS,t+1} = \frac{1}{(1+\mu)(1+\nu)}(1+r_t)(W_{LS,t} - Tr_{LS,t}). \tag{12}$$

3.4 Aggregation and Production

National wealth W_t is the sum of total private wealth, government net wealth $W_{G,t}$, and LSRA net wealth $W_{LS,t}$; and total labor supply L_t is measured in efficiency units:

$$W_t = \sum_{i=20}^{109} \int_{E\times A\times B} a \, \mathrm{d}X_t(\mathbf{s}) + W_{LS,t} + W_{G,t}, \tag{13}$$

$$L_t = \sum_{i=20}^{109} \int_{E\times A\times B} e \, h(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t) \, \mathrm{d}X_t(\mathbf{s}). \tag{14}$$

In a closed economy, capital stock is equal to national wealth, that is, $K_t = W_t$, and gross national product Y_t is determined by a constant-returns-to-scale production function,

$$Y_t = F(K_t, L_t).$$

The profit-maximizing condition for this competitive firm is

$$F_K(K_t, L_t) = r_t + \delta, \tag{15}$$

$$F_L(K_t, L_t) = w_t, \tag{16}$$

where δ is the depreciation rate of capital.

In a small open economy, factor prices, r_t^* and w_t^* are fixed at international levels as international capital flows assure that the capital-labor ratio determined by the world interest rate is attained in this country. The domestic capital stock $K_{D,t}$ and labor supply L_t, therefore, are determined so that the firm's profit maximizing condition satisfies,

$$F_K(K_{D,t}, L_t) = r_t^* + \delta,$$

$$F_L(K_{D,t}, L_t) = w_t^*.$$

Gross domestic product $Y_{D,t}$ is determined by the production function,

$$Y_{D,t} = F(K_{D,t}, L_t),$$

and gross national product Y_t is determined by

$$Y_t = (r_t^* + \delta)W_t + w_t^* \, L_t.$$

3.5 Recursive Competitive Equilibrium

***Definition* Recursive Competitive Equilibrium.** Let $\mathbf{s} = (i, e, a)$ be the individual state of households, let $\mathbf{S}_t = (x_t(\mathbf{s}), W_{LS,t}, W_{G,t})$ be the state of the economy, and let $\mathbf{\Psi}_t$ be the government policy schedule known at the beginning of year t,

$$\mathbf{\Psi}_t = \{W_{LS,s+1}, W_{G,s+1}, C_{G,s}, \tau_{I,s}(.), \tau_{P,s}(.), tr_{SS,s}(\mathbf{s}), tr_{LS,s}\}_{s=t}^{\infty}.$$

A series of factor prices $\{r_s, w_s\}_{s=t}^{\infty}$, accidental bequests $\{q_s\}_{s=t}^{\infty}$, the policy variables $\{W_{LS,s+1}, W_{G,s+1}, C_{G,s}, tr_{LS,s}(\mathbf{s})\}_{s=t}^{\infty}$, the parameters of policy functions $\{\varphi_s\}_{s=t}^{\infty}$, the value function of households $\{v(\mathbf{s}, \mathbf{S}_s; \mathbf{\Psi}_s)\}_{s=t}^{\infty}$, the decision rule of households

$$\{\mathbf{d}(\mathbf{s}, \mathbf{S}_s; \mathbf{\Psi}_s)\}_{s=t}^{\infty} = \{c(\mathbf{s}, \mathbf{S}_s; \mathbf{\Psi}_s), h(\mathbf{s}, \mathbf{S}_s; \mathbf{\Psi}_s), a'(\mathbf{s}, \mathbf{S}_s; \mathbf{\Psi}_s)\}_{s=t}^{\infty},$$

and the measure of households $\{x_s(\mathbf{s})\}_{s=t}^{\infty}$, are in a recursive competitive equilibrium if, in every period $s = t, ..., \infty$, each household solves the utility maximization problem (1)–(5) taking $\mathbf{\Psi}_t$ as given; the firm solves its profit maximization problem, the capital and labor market conditions (13)–(16) clear, and the government policy schedule satisfies (6)–(12).

In steady-state,

$$\mathbf{S}_{t+1} = \mathbf{S}_t$$

for all t and $\mathbf{s} \in I \times E \times A$.

4 Calibration

The parameters used for the 90-period OLG model are summarized in Table 5 and Table 6. We use the Cobb-Douglas utility function with constant relative risk aversion. The coefficient of relative risk aversion, γ is assumed to be a relatively high value, 3.0, to emphasize the effect of annuity markets against lifetime uncertainty. The maximum working hours of a household (married couple), $h_{\max}$, is set at 8,760, equal to 12 hours per day per person $\times$ 365 days $\times$ two persons. The share parameter of consumption α is chosen so that the average working hours of households between ages 20 and 64 equals 3,576 hours in the initial steady-state economy, the average number of hours supplied by married households in the 2003 PSID.

The working ability in Table 7 corresponds to the hourly wage of each household in the 2003 PSID.

We calculate the average hourly wages of 5-year age cohorts in 8 classes from the 2003 PSID and convert those into the 2001 wages, using the average hourly earnings growth of production workers. Then, we apply a shape-preserving cubic spline interpolation between each five-year age cohort to obtain the working ability for each age cohort. In a representative-agent model economy, we only use the 40-60th percentile wage of each age. (The age-working ability schedule of a heterogeneous-agent model will be discussed in the next version of this manuscript.)

The population growth rate ν is set to one percent per year while the survival rate ϕ_i at the end of age $i = \{20, ..., 109\}$ are the weighted averages of the male and female survival rates, as calculated by the Social Security Administration (2001). The survival rates at the end of age 109 are replaced by zero, thereby capping the maximum length of life.

The time preference parameter β is chosen in each variant of our model explored below so that the capital-GDP ratio in the initial steady state economy is 2.74, the empirical value in 2000. Production takes the Cobb-Douglas form,

$$F(K_t, L_t) = A_t K_t^{\theta} L_t^{1-\theta}.$$

where, recall, L_t is the sum of working hours in efficiency units. The capital share of output is given by

$$\theta = 1 - \frac{\text{Compensation of Employees} + (1-\theta) \times \text{Proprietors' Income}}{\text{National Income} + \text{Consumption of Fixed Capital}}.$$

The value of θ in 2000 was 0.30. The annual per-capita growth rate μ is assumed to be 1.8 percent. Total factor productivity A is set at 0.949, which normalizes the wage(per efficient labor unit) to unity.

The depreciation rate of fixed capital $\delta$is chosen by the following steady-state condition,

$$\delta = \frac{\text{Total Gross Investment}}{\text{Fixed Capital}} - \mu - \nu.$$

In 2000, private gross fixed investment accounted for 17.2 percent of GDP, and government (federal and state) gross investment accounted for 3.3 percent of GDP. With a capital-output ratio of 2.74, the ratio of gross investment to fixed capital is 7.5 percent. Subtracting productivity and population growth rates, the annual depreciation rate is 4.7 percent.

Federal income tax and state and local taxes are assumed to be at the level in year 2001 before the passage of the "Economic Growth and Tax Relief Reconciliation Act of 2001" (EGTRRA). Since households in our model are assumed to be married, we use a standard deduction of $7,600. The additional exemption per dependent person is $2,900. Table 8 shows the statutory marginal tax rates before EGTRRA. As noted earlier, a household's labor income in this calibration includes the imputed payroll tax paid by its employer. Thus, taxable income is obtained by subtracting the employer portion of payroll tax from labor income. The standard deduction, the personal exemption, and all tax brackets grow with productivity over time so that there is no real bracket creep; this indexing is also needed for the initial economy to be in steady state.

The tax rate levied on employees for Old-Age, Survivors, and Disability Insurance (OASDI) is 12.4 percent, and the tax rate for Medicare (HI) is 2.9 percent. In 2001, employee compensation above $80,400 was not taxable for OASDI. (See Table 9.) Workers with wages above $80,400, therefore, do not face a marginal tax or distortions from the Social Security system.

5 Introducing Perfect Annuity Markets

In this section, we show the macroeconomic and welfare effects of introducing perfect annuity markets to the economy without those. Actuarially fair annuity markets are introduced in year 1, and new equilibrium transition paths are obtained under several different assumptions. The introduction of perfect annuity markets affects the government budget—the Social Security budget and possibly the rest of the government budget—through the changes in labor supply and saving. Here, the government budget is assumed to be balanced by introducing a flat tax (possibly negative) on labor income and capital income.

The introduction of perfect annuity markets eliminates precautionary saving motives against lifetime uncertainty and decreases national wealth. Labor supply tends to increase slightly by the (negative) income effect of the elimination of accidental bequests. However, households near retirement decrease labor supply because the optimal wealth level at the time of retirement is lowered in the presence of annuity markets. National output (GNP) may increase or decrease in the short run but decreases in the most of the transition path. In a closed economy the interest rate rises and the wage rate falls.

5.1 Representative-Agent Economy without Wage Shocks

The first two panels of Table 10 (Runs 1-2) show the macroeconomic effect of perfect annuity markets when accidental bequests in the initial economy are uniformly and deterministically inherited by working-age households. These runs also assume that there is no progressive income tax other than the Social Security payroll tax in the economy. Under the closed economy assumption, GNP decreases by 2.1% in year 20 compared to the baseline economy without annuity markets, and it decreases by

2.4% in the long run. In a small open economy, since factor prices are kept at the same level as those in the baseline economy, national wealth decreases more and labor supply increases more. GNP decreases 5.4% by year 20 and 6.8% in the long run.

The first two panels of Table 11 (Runs 5-6) also show the macroeconomic effect in the economy with progressive income tax. The percent decrease in GNP is slightly larger in a closed economy with an income tax than that without an income tax, but the percent decrease is slightly smaller in a small open economy.

The welfare gains or losses are measured in terms of compensating variations in wealth, i.e., the negative of transfers required to restore the welfare level of the pre-reform economy. The overall efficiency gain or loss from the policy change is measured by the uniform transfer the lump-sum redistribution authority (LSRA) makes to each of the future households that enter the economy after the policy change.

When accidental bequests are assumed to be distributed uniformly to all working-age households every year, the introduction of perfect annuity markets makes the current middle-age and elderly households better off and the current young and future households worse off. The former group of households is benefited from new annuity markets that stabilize their consumption paths. The latter households tend to be hurt by the elimination of inheritance and the reduction in the wage rate.

The first panel of Table 12 shows the welfare effects of Runs 1-2 and 5-6. Households aged 60 at the time of policy change are better off by $92,400 in a closed economy without an income tax and by $77,900 in small-open economy, but households aged 20 at the time of policy change are worse off by $34,500 and $41,800 in a closed economy and small-open economy, respectively. Table 12 also shows the overall efficiency changes calculated with LSRA. Introducing perfect annuity markets generates efficiency loss of $1,100 per future household in a closed economy and $1,300 in a small-open economy. If we assume the economy with a progressive income tax, the efficiency results are slightly better in both closed economy and small-open economy.

The next two panels of Table 10 and Table 11 (Runs 3-4 and 7-8) show the macroeconomic effects of perfect annuity markets when stochastic inheritance of accidental bequests are assumed. Under this assumption, the reduction in national wealth is smaller in both closed and small-open economies, and the reduction in GNP is smaller slightly throughout the transition path, compared to the economies with deterministic inheritance.

The second panel of Table 12 shows the corresponding welfare and efficiency effects. In the economy without a progressive income tax (Runs 3-4), the welfare gains of elderly households at the time of policy change tend to be smaller but the welfare losses of young and future households also tend to be smaller. The overall efficiency gain calculated with LSRA is $5,200 in a closed economy and $4,900 in a small open economy. Because annuity markets would eliminate the inheritance shocks and eliminate the inequality in wealth, we can expect efficiency gains by introducing annuity markets.

In the economy with a progressive income tax (Runs 7-8), however, the reduction in national wealth is lager, and so is the reduction of GNP. The overall efficiency effects are ambiguous. The introduction of perfect annuity markets generates the efficiency gain of $700 per future household in a closed economy but the loss of $1,100 in a small open economy. In sum, in a realistic economy with a progressive income tax, the efficiency effect of annuity markets is very small and its sign is ambiguous in a representative-agent OLG economy.

5.2 Heterogeneous-Agent Economy with Wage Shocks

[TO BE COMPLETED.]

6 Privatizing Social Security

In this section, we analyze a stylized phased-in partial privatization of Social Security that begins in year 1, which is similar to the one in Nishiyama and Smetters (2007). Statutory Social Security benefits are reduced linearly over time cohort by cohort. Households age 65 and older in year 1 receive the current-law benefits throughout the rest of their lifetime; households of age 65 in year 2 receive benefits that are 1.25% lower than the current-law level throughout the rest of lifetime; households of age 65 in year 3 receive benefits 2.5% lower than the current-law level, and so on. Households of age 25 or younger in year 1 receive one half of their traditional Social Security benefits when they turn 65.

Pay-as-you-go payroll taxes are also reduced over time, but the effective tax rates on working-age households at the time of the reform actually increase during the transition. Because these workers help pay for the transitional benefits owed to retirees but do not receive full benefits themselves. Workers born in the long run, however, enjoy lower effective tax rates on their labor income.

When we assume a progressive income tax (in addition to the Social Security payroll tax) and government expenditure, the privatization of social security affects the rest of the government budget through the changes in labor supply and saving. The rest of the government budget is balanced by introducing a flat tax (possibly negative) on labor income and capital income.

For simplicity, assets in the new private accounts are assumed to be perfect substitutes with other private assets, including earning the same market rate of return and being subject to the same income tax schedule, as outlined above. As a result, the new private accounts do not have to be explicitly modeled, and households will increase their savings in response to a decline in retirement benefits.

6.1 Representative-Agent Economy without Wage Shocks

Table 13 shows the macroeconomic effects of Social Security privatization in a representative-agent economy without a progressive income tax, Table 14 shows the macroeconomic effects in the economy with a progressive income tax, and Table 15 shows the changes in welfare per household in economies with and without an income tax.

The first two panels in Table 13and Table 14 and the first panel in Table 15 (Runs 9-10 and 15-16) assume the economy with perfect annuity markets before and after the Social Security privatization.

Since the traditional pay-as-you-go Social Security is partially replaced with private accounts, the unfunded liability of the government is reduced, and national wealth is increased throughout the transition path. Labor supply is also increased because the distortion from the Social Security payroll tax is on average reduced by the privatization, even though the current and near-future working-age households bear the transition costs. Accordingly, national output (GNP) also increases throughout the transition path after the Social security privatization.

Except for the first few years, the percent increase in national wealth is larger than that in labor supply. So, the interest rate falls down and the wage rate goes up in the transition path. The Social Security payroll tax rate is eventually reduced to less than half of the pre-reform level, but, the payroll tax is higher than the half of the pre-reform level for more than 40 years because of the transition cost.

The welfare changes of retired households aged 65 and older, in year 1 are small, but working-age households in year 1 and in the near future are worse off, because they need to pay larger payroll taxes relative to their future benefits. However, future households who enter the economy after a couple of decades are better off from the privatization because the distortion from the Social Security payroll tax is reduced.

In a closed economy without a progressive income tax (Run 9), GNP increases by 2.5% in year 20 and 7.1% in the long run from the baseline (pre-reform) economy. The welfare loss (measured by compensating variations in one-time wealth transfer) of an age-40 household in year 1 is $44,300, the welfare gain of an age-0 household (a household who enters the economy in year 21) is $8,300, and the welfare gain of a future household converges around $46,400. When the Lump-Sum Redistribution Authority (LSRA) is turned on, each future household will be better off by $100, i.e., the partial Social Security privatization generates a very small efficiency gain in the economy.

In a small-open economy without an income tax (Run 10), the increase in national wealth is much larger, and the increase in labor supply is larger in the near future but smaller in the long run. GNP increases by 4.3 percent in year 20 and 12.6 percent in the long run. Welfare losses of current working-age households are significantly smaller than those in a closed economy, and welfare gains of future households are about the same. Thus, we might be able to expect that the privatization would generate a larger efficiency gain in a small-open economy. This simple aggregation is wrong, however. The privatization actually generates an efficiency loss of $3,700 per future household in a small-open economy. Because the LSRA transfers to the current households generate a sizable government debt, the interest rate is significantly higher with the LSRA turned on than that without it in a closed economy, and the higher interest rate reduces the necessary transfers to compensate the welfare loss of current households.

In a closed economy with progressive income tax (Run 15), the welfare loss and gain of the current and future households are both larger than those in the economy without an in come tax. Current working-age households suffer more because the distortion from the payroll tax to cover the transition costs is larger in the presence of an income tax, and the future households gain more because of the reduction of more distortional payroll tax for the Social Security pension. In total, however, the privatization of Social Security generates a significantly larger efficiency gain, which is $32,600 per future household when the LSRA is turned on.

The next two panels of Table 13 and Table 14 and the second panel of Table 15 (Runs 11-12 and 17-18) assume no perfect annuity markets in the economy and that the wealth left by deceased households are inherited (distributed) uniformly and deterministically by all working-age households.

In the absence of perfect annuity markets, partial privatization of Social Security reduces the insurance against lifetime uncertainty and increases risk. Thus, welfare losses of current households from the privatization are significantly larger than those in the economy with annuity markets. However, privatization increases lifecycle (retirement) saving and, accordingly, accidental bequests. So, welfare gains of future households are also larger, because they can receive higher wages and inherit larger bequests.

In a closed economy without annuity markets and an income tax, the efficiency loss from privatization is $5,600 per future household, which is significantly worse than the efficiency gain of $100 in the economy with annuity markets. This is not the case, however, when we assume a progressive income tax in an economy without perfect annuity markets. The overall efficiency gain from privatization is $41,500 per future household, which is much larger than the gain, $32,600, in the economy with annuity markets.

Why do annuity markets generate negative and positive marginal effects in economies with and without a progressive income tax? A possible explanation is the externality of precautionary savings through a distortional capital income tax. The distortion caused by the capital income tax is positively related to the interest-rate elasticity of saving of households. In the absence of perfect annuity markets, Social Security privatization generates a larger increase in wealth due to the precautionary saving against lifetime uncertainty. When a larger portion of saving is precautionary, the interest-rate elasticity of saving is lower, so is the distortion from the capital income tax. In addition, a larger wealth and a larger capital income allow the government to reduce marginal income tax rates, given that government expenditure other than Social Security is unchanged, and the overall distortion is reduced further.

The efficiency effect of annuity markets is under-estimated in the model economy with uniform and deterministic inheritance of accidental bequests, because in the economy working-age households receive a stable flow of transfers from the earlier generations in the absence of annuity markets. The third panel of Table 15 (Runs 13-14 and 19-20) shows the welfare changes in the economy with stochastic inheritance. As outlined above, in each year only a small portion of working-age households inherit accidental bequests.

In a closed economy without an income tax, the overall efficiency loss from the privatization is $8,700 per future household, and it is worse than the loss of $5,600 in the economy with deterministic inheritance. In a closed economy with an income tax, the efficiency gain is $30,600 per future household, and it is also worse than the gain of $41,500 in the economy with deterministic inheritance. Comparing the last result (Run 19) with the one in the economy with perfect annuity markets (Run 15), we observe that the social security privatization is better in the presence of perfect annuity markets if accidental bequests are unequal enough.

6.2 Heterogeneous-Agent Economy with Wage Shocks

In the deterministic and representative-agent economy, accidental bequests (with stochastic inheritance) are the only source of risk and inequality in wealth. Thus, annuity markets tend to have a positive efficiency effect. If the households' labor productivity is also stochastic and heterogeneous, however, the re-distribution with accidental bequests may increase or decrease the overall risk and inequality. We plan to analyze the efficiency effect of perfect annuity market in a heterogeneous-agent OLG economy, which is realistically calibrated to the U.S. economy. [TO BE COMPLETED.]

7 Concluding Remarks

This paper analyses the overall efficiency gains or losses from introducing perfect annuity markets and privatizing Social Security pension. To do so, we use the mechanism of the Lump-Sum Redistribution Authority originally used in Auerbach and Kotlikoff (1987).

In the presence of perfect annuity markets, macroeconomic variables like national wealth and GNP are always smaller than those in the absence of perfect annuity markets. In an economy with a progressive income tax, moreover, the efficiency effect of introducing annuity markets is very small or possibly negative. Also, though it is counter-intuitive, the overall efficiency gain from the Social Security privatization will likely be smaller in the economy with private annuity markets, because of the externality effect through a distortional income tax.

Regarding the last mentioned policy change, however, the welfare losses of current working-age households are relatively small if the introduction of private accounts is accompanied with perfect annuity markets. Thus, providing annuity options with the Social Security reform will be desirable if the current and near future households must bear the transition cost of the privatization and if the government does not have any policy tool to restore the welfare losses of those households.

In our model economy, for simplicity, each household is a married couple without any altruistic bequest motives, and a husband and a wife die at the same time (in the same year). Thus, the positive insurance effect of annuity markets is over emphasized in this paper. However, if we allowed households to share the risk of uncertain lifetime, the efficiency effect of perfect annuity markets would be even smaller or worse, and the implications of this paper would be strengthen.

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Table 1. The Effect of Annuity Markets When $w = 1$, $r = 0$ and $\beta = 1$

	Without Annuity Markets				WithAnnuity Markets					
	Baseline Economy				Long Run			Transition		
ϕ	c_1^*	c_2^*	b	Eu^*	c_i^{**}	Eu^{**}	diff.	c_i^{**}	Eu^{**}	diff.
0.1	0.990	0.099	0.089	-0.105	0.909	-0.046	0.059	0.990	-0.005	0.100
0.2	0.962	0.192	0.154	-0.160	0.833	-0.095	0.065	0.962	-0.020	0.140
0.3	0.917	0.275	0.193	-0.206	0.769	-0.148	0.057	0.917	-0.049	0.157
0.4	0.862	0.345	0.207	-0.249	0.714	-0.205	0.045	0.862	-0.090	0.159
0.5	0.800	0.400	0.200	-0.296	0.667	-0.264	0.032	0.800	-0.145	0.151
0.6	0.735	0.441	0.176	-0.347	0.625	-0.327	0.020	0.735	-0.214	0.133
0.7	0.671	0.470	0.141	-0.403	0.588	-0.392	0.011	0.671	-0.294	0.108
0.8	0.610	0.488	0.098	-0.464	0.556	-0.459	0.005	0.610	-0.387	0.078
0.9	0.552	0.497	0.050	-0.531	0.526	-0.530	0.001	0.552	-0.490	0.041
1.0	0.500	0.500	0.000	-0.602	0.500	-0.602	0.000	0.500	-0.602	0.000

Table 2. The Effect of Annuity Markets When $w = 1$, $r = 0.5$ and $\beta = 2/3$

	Without Annuity Markets				With Annuity Markets					
	Baseline Economy				Long Run			Transition		
ϕ	c_1^*	c_2^*	b	Eu^*	c_i^{**}	Eu^{**}	diff.	c_i^{**}	Eu^{**}	diff.
0.1	1.024	0.102	0.061	-0.056	0.938	-0.030	0.026	0.995	-0.002	0.053
0.2	1.027	0.205	0.110	-0.080	0.882	-0.062	0.018	0.979	-0.010	0.069
0.3	1.010	0.303	0.141	-0.099	0.833	-0.095	0.004	0.951	-0.026	0.073
0.4	0.974	0.390	0.156	-0.121	0.789	-0.130	-0.009	0.913	-0.050	0.070
0.5	0.923	0.462	0.154	-0.147	0.750	-0.167	-0.020	0.865	-0.084	0.063
0.6	0.862	0.517	0.138	-0.179	0.714	-0.205	-0.026	0.813	-0.126	0.053
0.7	0.796	0.557	0.111	-0.218	0.682	-0.244	-0.026	0.758	-0.177	0.041
0.8	0.728	0.583	0.078	-0.263	0.652	-0.285	-0.022	0.703	-0.235	0.028
0.9	0.662	0.596	0.040	-0.314	0.625	-0.327	-0.013	0.650	-0.300	0.014
1.0	0.600	0.600	0.000	-0.370	0.600	-0.370	0.000	0.600	-0.370	0.000

Table 3. The Effect of Annuity Markets When $w = 1$, $r = 1$ and $\beta = 0.5$

	Without Annuity Markets				With Annuity Markets					
	Baseline Economy				Long Run			Transition		
ϕ	c_1^*	c_2^*	b	Eu^*	c_i^{**}	Eu^{**}	diff.	c_i^{**}	Eu^{**}	diff.
0.1	1.042	0.104	0.047	-0.031	0.952	-0.022	0.009	0.997	-0.001	0.030
0.2	1.064	0.213	0.085	-0.040	0.909	-0.046	-0.005	0.986	-0.007	0.034
0.3	1.064	0.319	0.112	-0.048	0.870	-0.070	-0.022	0.967	-0.017	0.031
0.4	1.042	0.417	0.125	-0.058	0.833	-0.095	-0.037	0.938	-0.034	0.025
0.5	1.000	0.500	0.125	-0.075	0.800	-0.121	-0.046	0.900	-0.057	0.018
0.6	0.943	0.566	0.113	-0.099	0.769	-0.148	-0.049	0.856	-0.088	0.012
0.7	0.877	0.614	0.092	-0.131	0.741	-0.176	-0.045	0.809	-0.124	0.007
0.8	0.806	0.645	0.065	-0.170	0.714	-0.205	-0.035	0.760	-0.167	0.003
0.9	0.735	0.662	0.033	-0.214	0.690	-0.234	-0.020	0.712	-0.213	0.001
1.0	0.667	0.667	0.000	-0.264	0.667	-0.264	0.000	0.667	-0.264	0.000

Table 4. The Compensating Variations in Wealth When $w = 1$, $r = 0.5$ and $\beta = 2/3$

	$r = 0.5$ and $\beta = 2/3$			$r = 1.0$ and $\beta = 0.5$		
	Annuity Markets		Efficiency Gain	Annuity Markets		Efficiency Gain
ϕ	$tr_{CV,\infty}$	$tr_{CV,1}$	$-tr_{CV,1} - tr_{CV,\infty}/r$	$tr_{CV,\infty}$	$tr_{CV,1}$	$-tr_{CV,1} - tr_{CV,\infty}/r$
0.1	-0.054	-0.116	0.224	-0.020	-0.067	0.087
0.2	-0.036	-0.146	0.219	0.011	-0.074	0.063
0.3	-0.008	-0.150	0.166	0.046	-0.066	0.020
0.4	0.017	-0.139	0.104	0.073	-0.052	-0.021
0.5	0.035	-0.119	0.049	0.088	-0.037	-0.051
0.6	0.043	-0.095	0.009	0.090	-0.023	-0.067
0.7	0.042	-0.070	-0.014	0.080	-0.012	-0.067
0.8	0.033	-0.045	-0.022	0.059	-0.005	-0.054
0.9	0.019	-0.021	-0.016	0.032	-0.001	-0.031
1.0	0.000	0.000	0.000	0.000	0.000	0.000

Table 5. Parameters Independent of Model Assumptions

Coefficient of relative risk aversion	γ	3.0
Capital share of output	θ	0.30
Depreciation rate of capital stock	δ	0.047
Long-term real growth rate	μ	0.018
Population growth rate	ν	0.010
Total factor productivity[a]	A	0.949

a. Total factor productivity is chosen so that w equals 1.0.

Table 6. Parameters that Vary by Model Assumptions

		Representative-agent economy without wage shocks		
		With perfect annuity market	Without annuity market	
			Deterministic inheritance	Stochastic inheritance
Without income tax				
Time preference[a]	β	1.017	1.012	1.011
Consumption share[b]	α	0.428	0.427	0.433
Prob. of receiving bequests	η		1.000	0.0161
With income tax				
Time preference[a]	β	1.036	1.030	1.029
Consumption share[b]	α	0.470	0.476	0.476
Prob. of receiving bequests	η		1.000	0.0161

a. The capital-GDP ratio is targeted to be 2.74 (r = 6.25 percent) without annuity markets.
b. The average annual working hours are 3,576 per married couple when $h_{\max}$ = 8,760.

Table 7. Working Abilities of a Household (in U.S. Dollars per Hour)

	Percentile	Age cohorts					
		20-24	25-29	30-34	35-39	40-44	45-49
e^1	0-20th	6.59	6.79	7.23	7.58	6.63	7.06
e^2	20-40th	9.13	11.90	12.99	14.13	13.65	14.01
e^3	40-60th	11.13	15.15	17.63	19.43	18.76	19.84
e^4	60-80th	13.89	18.79	23.72	25.98	26.56	26.51
e^5	80-90th	17.89	23.07	31.94	36.66	37.30	34.38
e^6	90-95th	22.17	28.75	44.87	50.36	51.30	43.69
e^7	95-99th	28.92	37.02	70.45	90.33	74.86	76.14
e^8	99-100th	50.99	68.56	111.40	180.53	211.09	239.59
	Percentile	Age cohorts					
		50-54	55-59	60-64	65-69	70-74	75-79
e^1	0-20th	6.45	2.76	0.02	0.00	0.00	0.00
e^2	20-40th	14.02	11.90	4.54	0.01	0.00	0.00
e^3	40-60th	20.46	17.75	12.55	3.56	0.00	0.00
e^4	60-80th	27.89	25.24	20.40	12.35	1.64	0.35
e^5	80-90th	37.71	32.90	32.30	22.41	7.45	10.15
e^6	90-95th	47.60	43.79	42.47	34.78	12.52	20.57
e^7	95-99th	81.61	68.69	57.48	47.01	19.22	36.73
e^8	99-100th	247.47	443.14	89.02	101.28	100.08	51.30

Source: Authors' calculations from the 2003 PSID.

Table 8. Marginal Individual Income Tax Rates in 2001 (Married Household, Filed Jointly)

Taxable income			Marginal income tax rate (%)
\$0	–	\$45,200	$15.0 \times \varphi_I$
\$45,200	–	\$109,250	$28.0 \times \varphi_I$
\$109,250	–	\$166,500	$31.0 \times \varphi_I$
\$166,500	–	\$297,350	$36.0 \times \varphi_I$
\$297,350	–		$39.6 \times \varphi_I$

Table 9. Marginal Payroll Tax Rates in 2001

Taxable labor income per worker			Marginal tax rate (%)	
			OASDI	HI
\$0	–	\$80,400	$12.4 \times \varphi_P$	2.9
\$80,400	–		$0.0 \times \varphi_P$	2.9

Note: The payroll tax adjustment factor φ_P equals 1.0 in the baseline economy.

Table 10. Introducing Perfect Annuity Markets to the Representative-Agent Economy without Income Tax
(Percent Change in Selected Macro Variables Relative to Baseline)

Run #	Year t	Output (GNP)	National wealth	Labor Supply	Interest rate	Wage rate	Income tax rate[a]
1	1	0.2	0.0	0.2	0.3	-0.1	0.0
Deterministic	10	-1.2	-5.4	0.6	7.7	-1.8	0.1
inheritance	20	-2.1	-7.5	0.4	10.4	-2.4	0.2
(closed)	30	-2.4	-8.1	0.2	11.0	-2.6	0.2
	40	-2.4	-8.2	0.2	11.0	-2.6	0.2
	∞	-2.4	-8.1	0.2	11.0	-2.6	0.2
2	1	-1.4	0.0	-2.0	0.0	0.0	0.2
Deterministic	10	-3.7	-13.0	0.2	0.0	0.0	0.0
inheritance	20	-5.4	-21.5	1.4	0.0	0.0	-0.1
(small open)	30	-6.3	-25.5	2.0	0.0	0.0	-0.2
	40	-6.6	-26.9	2.2	0.0	0.0	-0.2
	∞	-6.8	-27.5	2.1	0.0	0.0	-0.2
3	1	-0.1	0.0	-0.1	-0.1	0.0	0.0
Stochastic	10	-1.2	-4.9	0.3	6.7	-1.6	0.1
inheritance	20	-2.0	-6.9	0.2	9.4	-2.2	0.2
(closed)	30	-2.3	-7.6	0.1	10.1	-2.4	0.2
	40	-2.3	-7.7	0.0	10.2	-2.4	0.2
	∞	-2.3	-7.7	0.1	10.3	-2.4	0.2
4	1	-1.5	0.0	-2.1	0.0	0.0	0.2
Stochastic	10	-3.6	-11.8	0.0	0.0	0.0	0.0
inheritance	20	-5.1	-19.8	1.2	0.0	0.0	-0.1
(small open)	30	-5.9	-23.6	1.7	0.0	0.0	-0.2
	40	-6.2	-25.1	1.9	0.0	0.0	-0.2
	∞	-6.4	-25.6	1.9	0.0	0.0	-0.2

a. The government budget is balanced by introducing flat labor and capital income tax. The numbers are the changes in percentage points.

Table 11. Introducing Perfect Annuity Markets to the Representative-Agent Economy with Income Tax
(Percent Change in Selected Macro Variables Relative to Baseline)

Run #	Year t	Output (GNP)	National wealth	Labor Supply	Interest rate	Wage rate	Income tax rate[a]
5	1	-0.2	0.0	-0.3	-0.4	0.1	0.1
Deterministic	10	-1.7	-5.4	-0.1	6.9	-1.6	0.5
inheritance	20	-2.7	-8.4	-0.2	10.8	-2.5	0.7
(closed)	30	-3.2	-9.7	-0.3	12.6	-2.9	0.9
	40	-3.5	-10.4	-0.4	13.4	-3.1	0.9
	∞	-3.7	-10.9	-0.4	14.2	-3.3	1.0
6	1	-1.9	0.0	-2.8	0.0	0.0	0.8
Deterministic	10	-3.8	-11.5	-0.4	0.0	0.0	0.7
inheritance	20	-5.1	-19.0	0.9	0.0	0.0	0.6
(small open)	30	-5.6	-22.5	1.6	0.0	0.0	0.5
	40	-5.9	-23.7	1.8	0.0	0.0	0.5
	∞	-6.0	-24.1	1.8	0.0	0.0	0.5
7	1	-0.4	0.0	-0.5	-0.6	0.2	0.1
Stochastic	10	-2.1	-6.1	-0.3	7.5	-1.8	0.6
inheritance	20	-3.3	-9.5	-0.5	12.1	-2.8	0.9
(closed)	30	-4.0	-11.2	-0.7	14.3	-3.3	1.1
	40	-4.3	-12.0	-0.8	15.3	-3.5	1.2
	∞	-4.5	-12.5	-0.8	16.2	-3.7	1.3
8	1	-2.2	0.0	-3.2	0.0	0.0	0.9
Stochastic	10	-4.3	-12.8	-0.7	0.0	0.0	0.9
inheritance	20	-5.8	-21.2	0.7	0.0	0.0	0.8
(small open)	30	-6.5	-25.2	1.5	0.0	0.0	0.8
	40	-6.8	-26.7	1.7	0.0	0.0	0.7
	∞	-7.0	-27.4	1.7	0.0	0.0	0.7

a. The government budget is balanced by introducing flat labor and capital income tax. The numbers are the changes in percentage points.

Table 12. Change in Welfare per Household When Introducing Perfect Annuity Markets to the Representative-Agent Economy
(Compensating Variations in Wealth, $1,000 in 2001)

	Age in year 1	Without LSRA				With LSRA			
		Without income tax		With income tax		Without income tax		With income tax	
		closed	open	closed	open	closed	open	closed	open
Run #		1	2	5	6	1	2	5	6
Baseline with deterministic inheritance	80	64.3	62.4	45.3	43.8	0.0	0.0	0.0	0.0
	60	92.4	77.9	84.2	69.4	0.0	0.0	0.0	0.0
	40	25.9	7.7	37.4	11.9	0.0	0.0	0.0	0.0
	20	-34.5	-41.8	-18.5	-37.8	0.0	0.0	0.0	0.0
	0	-42.4	-39.4	-27.7	-35.2	-1.1	-1.3	-0.5	0.8
	-20	-42.5	-39.2	-29.5	-34.6	-1.1	-1.3	-0.5	0.8
	-40	-42.6	-40.0	-29.9	-35.1	-1.1	-1.3	-0.5	0.8
	$-\infty$	-42.5	-39.9	-30.0	-35.3	-1.1	-1.3	-0.5	0.8
Run #		3	4	7	8	3	4	7	8
Baseline with stochastic inheritance	80	41.4	40.3	45.9	44.4	0.0	0.0	0.0	0.0
	60	80.7	68.5	86.0	70.2	0.0	0.0	0.0	0.0
	40	27.5	10.3	39.1	11.4	0.0	0.0	0.0	0.0
	20	-21.8	-27.0	-17.5	-35.2	0.0	0.0	0.0	0.0
	0	-26.7	-25.4	-27.5	-33.9	5.2	4.9	0.7	-1.1
	-20	-27.0	-25.2	-29.5	-33.6	5.2	4.9	0.7	-1.1
	-40	-27.0	-25.3	-29.8	-33.6	5.2	4.9	0.7	-1.1
	$-\infty$	-27.0	-25.3	-30.0	-33.8	5.2	4.9	0.7	-1.1

Table 13. Privatizing Social Security to the Representative-Agent Economy without Income Tax
(Percent Change in Selected Macro Variables Relative to Baseline)

Run #	Year t	Output (GNP)	National wealth	Labor Supply	Interest rate	Wage rate	Payroll tax rate[a]
9	1	0.4	0.0	0.6	0.8	-0.2	-0.5
With perfect	10	1.3	2.3	0.8	-1.8	0.4	-4.7
annuity	20	2.5	4.8	1.5	-3.9	1.0	-14.9
markets	30	3.9	7.6	2.3	-6.0	1.5	-27.8
(closed)	40	5.3	10.3	3.2	-8.0	2.0	-40.9
	∞	7.1	15.8	3.6	-13.0	3.4	-54.8
10	1	0.9	0.0	1.3	0.0	0.0	-1.4
With perfect	10	2.3	4.9	1.2	0.0	0.0	-4.6
annuity	20	4.3	10.9	1.4	0.0	0.0	-14.0
markets	30	6.5	17.4	1.8	0.0	0.0	-26.3
(small open)	40	8.6	23.8	2.2	0.0	0.0	-39.0
	∞	12.6	39.2	1.2	0.0	0.0	-52.1
11	1	-0.7	0.0	-0.9	-1.2	0.3	0.7
No annuity	10	0.2	2.2	-0.6	-3.4	0.8	-3.7
markets; with	20	1.6	5.3	0.1	-6.2	1.5	-14.2
deterministic	30	3.3	9.0	1.0	-9.1	2.3	-27.4
inheritance	40	4.9	12.6	1.8	-12.0	3.1	-40.7
(closed)	∞	7.2	19.8	2.2	-18.5	4.9	-54.8
12	1	0.0	0.0	0.0	0.0	0.0	0.0
No annuity	10	2.0	6.8	0.0	0.0	0.0	-3.4
markets; with	20	4.7	15.8	0.0	0.0	0.0	-12.7
deterministic	30	7.7	25.7	0.1	0.0	0.0	-24.9
inheritance	40	10.7	35.4	0.1	0.0	0.0	-37.7
(small open)	∞	17.4	63.6	-2.4	0.0	0.0	-50.3
13	1	0.6	0.0	0.9	1.1	-0.3	-0.6
No annuity	10	1.8	3.3	1.2	-2.6	0.6	-5.2
markets; with	20	3.4	7.0	1.9	-5.9	1.5	-15.7
stochastic	30	5.1	10.9	2.7	-9.1	2.3	-28.7
inheritance	40	6.8	14.7	3.5	-12.1	3.1	-41.7
(closed)	∞	9.1	22.3	3.9	-18.8	5.0	-55.7
14	1	1.5	0.0	2.1	0.0	0.0	-2.2
No annuity	10	3.7	7.8	1.9	0.0	0.0	-5.3
markets; with	20	6.7	17.8	1.9	0.0	0.0	-14.4
stochastic	30	10.0	28.7	1.9	0.0	0.0	-26.4
inheritance	40	13.2	39.5	1.9	0.0	0.0	-38.8
(small open)	∞	21.1	72.3	-0.8	0.0	0.0	-51.1

a. The Social Security budget is assumed to be pay-as-you-go, and it is balanced by the payroll tax rate.

Table 14. Privatizing Social Security to the Representative-Agent Economy
with Income Tax
(Percent Change in Selected Macro Variables Relative to Baseline)

Run #	Year t	Output (GNP)	National wealth	Labor supply	Interest rate	Wage rate	Payroll tax rate[a]	Income tax rate[b]
15	1	0.7	0.0	1.0	1.2	-0.3	-0.7	-0.2
With perfect	10	2.1	3.0	1.7	-1.6	0.4	-5.5	-0.6
annuity	20	4.2	7.0	3.0	-4.6	1.1	-16.4	-1.1
markets	30	6.7	11.7	4.6	-7.9	2.0	-29.8	-1.7
(closed)	40	9.2	16.7	6.1	-11.3	2.9	-43.0	-2.2
	∞	13.6	29.3	7.4	-21.3	5.7	-57.4	-3.1
16	1	1.3	0.0	1.9	0.0	0.0	-1.9	-0.4
With perfect	10	3.0	5.2	2.1	0.0	0.0	-5.5	-0.7
annuity	20	5.7	12.2	2.9	0.0	0.0	-15.3	-1.3
markets	30	8.9	20.3	4.0	0.0	0.0	-27.9	-1.9
(small open)	40	12.2	29.2	4.9	0.0	0.0	-40.7	-2.5
	∞	18.9	54.0	3.8	0.0	0.0	-53.3	-3.3
17	1	1.1	0.0	1.6	1.9	-0.5	-1.2	-0.4
No annuity	10	3.3	4.9	2.6	-2.7	0.7	-6.6	-0.9
markets; with	20	6.3	11.2	4.2	-7.8	2.0	-18.1	-1.6
deterministic	30	9.5	18.3	5.9	-13.1	3.4	-31.7	-2.2
inheritance	40	12.4	25.3	7.3	-18.0	4.8	-44.8	-2.8
(closed)	∞	17.7	42.3	8.5	-30.2	8.5	-59.0	-3.7
18	1	2.2	0.0	3.2	0.0	0.0	-3.3	-0.7
No annuity	10	5.1	9.2	3.4	0.0	0.0	-6.8	-1.2
markets; with	20	9.2	21.2	4.0	0.0	0.0	-16.3	-1.9
deterministic	30	13.6	34.7	4.6	0.0	0.0	-28.3	-2.6
inheritance	40	17.9	48.3	4.9	0.0	0.0	-40.7	-3.2
(small open)	∞	29.3	93.6	1.7	0.0	0.0	-52.3	-4.4
19	1	1.1	0.0	1.6	1.9	-0.5	-1.1	-0.3
No annuity	10	3.2	4.8	2.5	-2.7	0.7	-6.6	-0.8
markets; with	20	6.2	11.0	4.2	-7.6	1.9	-18.0	-1.5
stochastic	30	9.4	18.1	5.9	-12.9	3.3	-31.6	-2.2
inheritance	40	12.4	25.2	7.3	-17.9	4.7	-44.7	-2.8
(closed)	∞	17.7	42.3	8.5	-30.3	8.5	-59.0	-3.6
20	1	2.2	0.0	3.2	0.0	0.0	-3.3	-0.7
No annuity	10	5.1	9.1	3.4	0.0	0.0	-6.8	-1.2
markets; with	20	9.3	21.4	4.1	0.0	0.0	-16.3	-1.9
stochastic	30	13.9	35.3	4.7	0.0	0.0	-28.4	-2.6
inheritance	40	18.4	49.6	5.1	0.0	0.0	-40.7	-3.3
(small open)	∞	31.1	99.8	1.6	0.0	0.0	-52.3	-4.6

a. The Social Security budget is assumed to be pay-as-you-go, and it is balanced by the payroll tax rate.
b. The rest of the government budget is balanced by proportionally adjusting income tax.

Table 15. Change in Welfare per Household When Privatizing Social Security
Markets to the Representative-Agent Economy
(Compensating Variations in Wealth, $1,000 in 2001)

	Age in year 1	Without LSRA[a]				With LSRA[b]			
		Without income tax		With income tax		Without income tax		With income tax	
		closed	open	closed	open	closed	open	closed	open
Run #		9	10	15	16	9	10	15	16
With perfect annuity markets	80	-0.2	0.0	0.1	0.3	0.0	0.0	0.0	0.0
	60	-22.2	-17.2	-23.0	-16.4	0.0	0.0	0.0	0.0
	40	-44.3	-29.4	-58.4	-33.7	0.0	0.0	0.0	0.0
	20	-22.0	-7.8	-36.8	-0.3	0.0	0.0	0.0	0.0
	0	8.3	20.4	14.1	54.0	0.1	-3.7	32.6	24.1
	-20	35.7	42.1	57.6	90.4	0.1	-3.7	32.6	24.1
	-40	44.8	46.1	74.9	98.1	0.1	-3.7	32.6	24.1
	$-\infty$	46.4	45.9	80.1	98.7	0.1	-3.7	32.6	24.1
Run #		11	12	17	18	11	12	17	18
No annuity markets; with deterministic inheritance	80	-0.8	0.0	0.3	0.4	0.0	0.0	0.0	0.0
	60	-34.9	-25.6	-33.1	-23.9	0.0	0.0	0.0	0.0
	40	-72.8	-49.7	-94.5	-52.1	0.0	0.0	0.0	0.0
	20	-47.0	-17.3	-84.0	-3.4	0.0	0.0	0.0	0.0
	0	6.9	25.7	12.5	74.9	-5.6	-10.6	41.5	37.1
	-20	48.6	62.2	79.3	133.7	-5.6	-10.7	41.5	37.1
	-40	63.2	74.4	107.7	156.6	-5.6	-10.6	41.5	37.1
	$-\infty$	66.2	79.4	117.7	168.8	-5.6	-10.6	41.5	37.1
Run #		13	14	19	20	13	14	19	20
No annuity markets; with stochastic inheritance	80	-0.1	0.0	0.3	0.4	0.0	0.0	0.0	0.0
	60	-31.6	-25.2	-32.8	-23.8	0.0	0.0	0.0	0.0
	40	-69.3	-47.2	-93.7	-52.5	0.0	0.0	0.0	0.0
	20	-35.6	-15.3	-62.1	-5.6	0.0	0.0	0.0	0.0
	0	9.6	25.1	9.9	68.7	-8.7	-10.4	30.6	29.5
	-20	51.3	58.6	75.4	123.3	-8.7	-10.4	30.5	29.5
	-40	66.3	69.3	103.4	143.4	-8.7	-10.4	30.6	29.5
	$-\infty$	69.5	73.5	113.3	154.4	-8.7	-10.4	30.6	29.5